LEARNING CURVES

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UNIT MODELL, CRAWFORD MODELL

x	y
cycles	unit
1	2,132
2	1,789
3	1,588
4	1,54
5	1,575
6	

estimation

$y = ax^b$ or $\ln y = \ln a + b \ln x;$

$learning\ rate\ \% = 100\,(2)^b$

For example b= - 0,151 if learning rate is 90%

"If there is learning in the production process, the cost of some *doubled unit* equals the cost of the undoubled unit times the slope of the learning curve"

WRIGHT MODELL CUMULATIVE AVERAGE MODEL

x	y
cycles	Cumulative av.
1	2,132
2	1,961
3	1,836
4	1,762
5	1,725
6	

$CA_t = (Y_1 + Y_2 + \ldots Y_{t-1} + \ldots + Y_t)/t.$

estimation

$y = ax^b$ or $\ln y = \ln a + b \ln x;$

"If there is learning in the production process, the cumulative average cost of some doubled unit equals the cumulative average cost of the undoubled unit times the slope of the learning curve"

Repetitive works, Wrigth 1936

	Cumulative	
Cycles	Time	Average
1	**1**	**1**
2	**1,8**	**0,9**
3	2,54	0,85
4	**3,24**	**0,81**
5	3,91	0,78
6	4,57	0,76
7	5,21	0,74
8	**5,83**	**0,73**

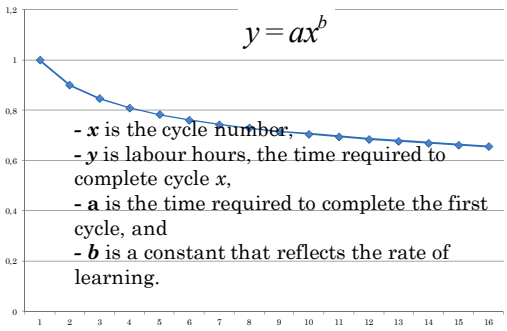
Learning Curve, Basics

$$y = ax^{\log_2 r}$$

- where
- „a" is the time required to complete the first cycle
- „y" is labour hours, the time required to complete cycle *x*
- „x" is the cycle number
- „r" rate of learning
- Since „x" monoton increasing, „r"<1, „a" pozitive, „y" monoton decreasing.

Learning curves

$$y = ax^b$$



- ***x*** is the cycle number,
- ***y*** is labour hours, the time required to complete cycle *x*,
- **a** is the time required to complete the first cycle, and
- ***b*** is a constant that reflects the rate of learning.

TRANSFORMATIONS
CUMULATIVE AVERAGE
MOVING AVERAGE
EXPONENTIAL AVERAGE

$$CA_t = (Y_1 + Y_2 + \ldots Y_{t-1} + \ldots + Y_t)/t.$$

$$\mathrm{MA_t} = (\mathrm{Y_t} + \mathrm{Y_{t-1}} + \mathrm{Y_{t-2}})/3$$

$$\mathrm{EA_t} = \alpha \mathrm{Y_t} + (1-\alpha)\mathrm{EA_{t-1}}$$

STANFORD B MODEL

x	y
cycles	
1	2,132
2	1,961
3	1,836
4	1,762
5	1,725
6	

estimation

$y = a(x+B)^b$ or $\ln y = \ln a + b\ln(\mathrm{x}+\mathrm{B})$;

Where ”B” 0-10 refers to the experience

DEJONG MODEL

x	y
cycles	
1	2,132
2	1,961
3	1,836
4	1,762
5	1,725
6	

estimation

$$y = a\left[M + \frac{1-M}{x^b}\right] = a_0 + (a-a_0)x^{-b}$$

Ahol „M” kompressziós együttható

S CURVE MODEL

x	y
cycles	
1	2,132
2	1,961
3	1,836
4	1,762
5	1,725
6	

estimation

$$y = a_0 + (a - a_0)(x + B)^b$$

DeJong and Stanford model in one

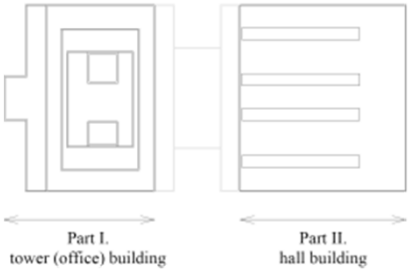
REPETITIVE WORKS, WRIGTH 1936

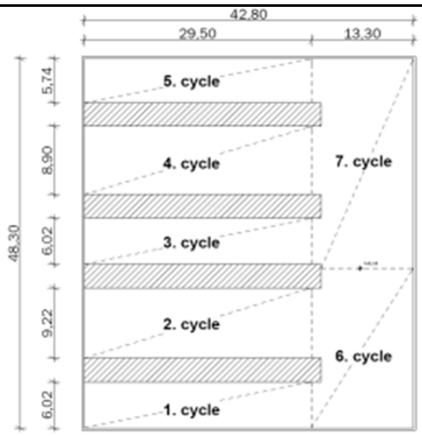
	Cumulative	
Cycles	Time	Average
1	**1**	**1**
2	**1,8**	**0,9**
3	2,54	0,85
4	**3,24**	**0,81**
5	3,91	0,78
6	4,57	0,76
7	5,21	0,74
8	**5,83**	**0,73**

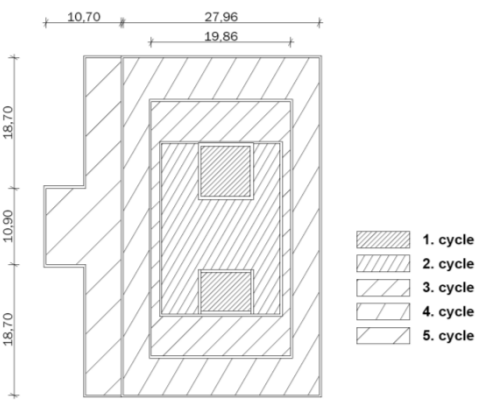
AIM

- Predict/Estimate the time required to complete a given cycle/product

RECONSTRUCTION WORK







Input Data

cycles	Unit	CA	MA	Exp. Av. 0,5
1	2,132	2,132	2,132	2,132
2	1,789	1,961	1,961	1,961
3	1,588	1,836	1,836	1,775
4	1,54	1,762	1,639	1,658
5	1,575	1,725	1,568	1,617
6	1,546	1,608	1,554	1,582
7	1,541	1,558	1,554	1,562

Estimation/Y values

cycles	Original	UNIT	CA	MA	EA; 0,5
1	2,132	2,084	2,137	2,184	2,161
2	1,789	1,807	1,946	1,911	1,908
3	1,588	1,663	1,842	1,767	1,773
4	1,54	1,567	1,771	1,671	1,684
5	1,575	1,497	1,718	1,601	1,617
6	1,546	1,442	1,676	1,546	1,565
7	1,541	1,397	1,642	1,5	1,522

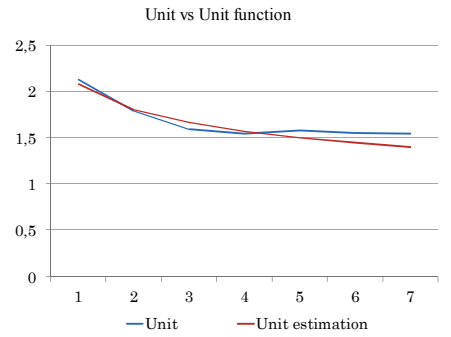
Results for Hall

cycles	UNIT	CA	MA	EA; 0,3	EA; 0,5
1	0,048	0,005	0,052	0,038	0,029
2	0,018	0,015	0,05	0,051	0,053
3	0,075	0,006	0,069	0,024	0,002
4	0,027	0,009	0,032	0,013	0,026
5	0,078	0,007	0,033	0,024	0
6	0,104	0,068	0,008	0,035	0,017
7	0,144	0,084	0,054	0,039	0,04
Accuracy 1-5	0,246	0,042	0,236	0,15	0,11
Accuracy 6-7	0,248	0,05	0,062	0,074	0,057

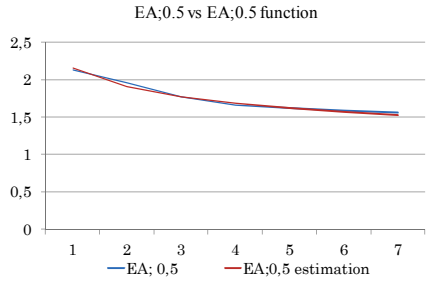
RESULTS FOR TOWER

cycles	UNIT	CA	MA	WMA	EA; 0,5
1	0,122	0,009	0,088	0,074	0,052
2	0,144	0,012	0,062	0,082	0,065
3	0,061	0,01	0,144	0,057	0,023
4	0,025	0,002	0,058	0	0,008
5	0,069	0,009	0,058	0,063	0,03
Accuracy 1-5	0,421	0,042	0,41	0,276	0,178

RESULT



RESULT



CONCLUSION

- Original time of cyle 6 =1,546
- Estimation based on EA: Y_6=1,513
- Estimation based on Unit: Y_6=1,442
- Original time of cyle 7 =1,541
- Estimation based on EA: Y_7=1,479
- Estimation based on Unit: Y_7=1,379

- Transform data to an average brings better prediction

CONCLUSION

- Mathematical model
  - linear log x log y method is the most precise predictor
- Data presentation methods
  - cumulative average and exponential average were very reliable predictors
- Futher research
  - investigating more complex functions
  - involve more data

THANKS FOR YOUR ATTENTION