Multiattribute decision making

Mályusz Levente
Department of Construction Technology and Management

Problem / raising questions

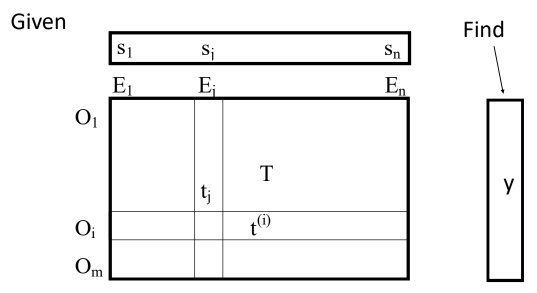
Given:
- Objects and know their properties, Object: house, plot, offer, plan ....
- Features/attributes: cost of construction, running costs, deadline ...
- Let's put the objects in some order of importance

Methods

- Pairwise comparison?
- Multicriteria optimization?
- Multiattribute Optimization?

Evaluating steps

- I **select objects to compare**,
- II. **defining the evaluation factors and their weights**, which can usually be achieved by organizing the evaluation factors by tree structure,
- III. **then the objects must be evaluated according to the evaluation factors**, ie the value of each object must be determined according to each evaluation factor,
- IV. **selecting the evaluation procedure/method** and carrying out the evaluation,

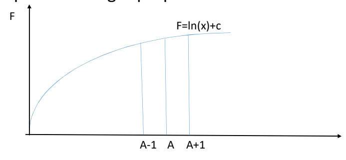


Scales 1.

- Nominal scale: dwelling house, not quantifiable, only frequency of occurrence can be investigated
- Ordinary Scale: Grades, "Better" or "Multiple" are the catchy (median, mean?)
- Interval Scale: Temperature, Time; the difference can be grasped, the ratio does not make sense Scale: consumption, Kelvin (temperature scale)
- Ratio scale: consumption (Kelvin temperature scale)

Daniel Bernoulli : Risk aversion

- Assumption: feeling is proportional to a relative increase in wealth

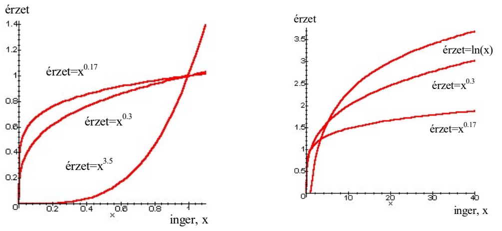


F(A+t)-F(A)>F(a)-F(a-t)

physical stimulus and the *perceived* change

- Earnst Heinrich Weber (1795-1878) quantify human response to a physical stimulus
- Weber–Fechner law: logaritmic function, f=const1*ln(x)+const2
- Stanley Smith Stevens, 1906-1973,
- Stevens's power law: power function, f=const*x$^{\alpha}$

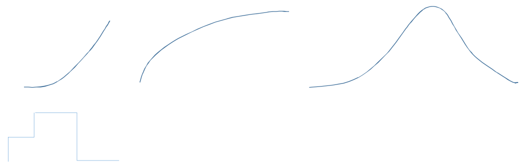
Divergence functions, „sensation function"



Stevens's law…

Continuum	Exponent (a {\displaystyle a})	Stimulus condition
Loudness	0.67	Sound pressure of 3000 Hz tone
Vibration	0.95	Amplitude of 60 Hz on finger
Vibration	0.6	Amplitude of 250 Hz on finger
Brightness	0.33	5° target in dark
Brightness	0.5	Point source
Brightness	0.5	Brief flash
Brightness	1	Point source briefly flashed
Lightness	1.2	Reflectance of gray papers
Visual length	1	Projected line
Visual area	0.7	Projected square
Redness (saturation)	1.7	Red-gray mixture
Taste	1.3	Sucrose
Taste	1.4	Salt
Taste	0.8	Saccharine
Smell	0.6	Heptane

Utility Functions



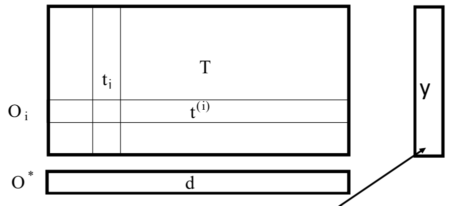
- Transform every attributes to 1-5, 0-10, 0-100, 1-10, 1-100 scales

Scales 3. 1-10 or 1-100…………………

1-10 vagy 1-100?

	T_1.	T_2.	
O_1.	16	26	42
O_2.	24	19	43
O_1.	2	3	5
O_2.	2	2	4

Dominance method



$$D\left(O_1 II O^*\right) = \sum_{j=1}^{n} D\left(t_j^{(1)} II d_j\right)$$

Divergence functions: Euclidean Difference

- Definition: Divergence the vector of **a**=(a_1,...,a_j,...,a_n) from vector **b**=(b_1,...,b_2,...b_n) is the value of the following expression:

$$D(\underline{a} II \underline{b}) = \sum_{j=1}^{n} \left((a_1 - b_1)^2 + (a_2 - b_2)^2 \right)$$

1. D(**a**II**b**)≥0,
2. D(**a**II**b**)=0 if and only if, ha **a**=**b**.
3. D(**a**II**b**)=D(**b**II**a**)

Divergence functions: Kullback-Leibler

- Divergence the vector of **a**=(a_1,...,a_j,...,a_n) >0 from vector **b**=(b_1,...,b_2,...b_n) >0 is the value of the following expression:

$$D(\underline{a} II \underline{b}) = \sum_{j=1}^{n} \left(a_j \ln \frac{a_j}{b_j} - a_j + b_j \right)$$

1. D(**a**II**b**)≥0,
2. D(**a**II**b**)=0 if and only if **a**=**b**.
3. D(**a**II**b**)≠D(**b**II**a**)

A priori (prior to the event), a posteriori (after the event)

- The vector „a” is after the "a posteriori" event
  Vector „b” in a pre-event (prior to the event) role

$$D(\underline{a}II\underline{b}) = \sum_{j=1}^{n}\left(a_j \ln\frac{a_j}{b_j} - a_j + b_j\right)$$

After event, fact

Prior to the event notion

Dominance method

	E_1.	E_2.	E_3.	E_4.	E_5.	E_6.
O_1.	1	2	4	2	1	4
O_2.	2	3	3	2	1	1
O_3.	3	3	2	3	4	1
O_4.	4	2	1	1	3	2
dominráns (d)	4	3	4	3	4	4

$$\sum_{j=1}^{n} D\left(t_j^{(1)} IId_j\right) = 1\ln\frac{1}{4} - 1 + 4 + 2\ln\frac{2}{3} - 2 + 3 + 4\ln\frac{4}{4} - 4 + 4 +$$
$$+2\ln\frac{2}{3} - 2 + 3 + 1\ln\frac{1}{4} - 1 + 4 + 4\ln\frac{4}{4} - 4 + 4 = 3.605$$

$y_1=D(t^{(1)}IId)=3.605$	
$y_2=D(t^{(2)}IId)=4.167$	
$y_3=D(t^{(3)}IId)=2.364$	
$y_4=D(t^{(4)}IId)=3.454$	

Hellinger, Pearson, Fischer divergence functions

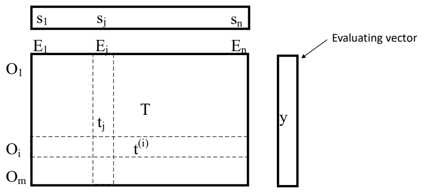
$$\sum_{j=1}^{n} a_j = 1 \quad \sum_{j=1}^{n} b_j = 1$$

$$D_H(\underline{a}II\underline{b}) = \sum_{j=1}^{n}\left(\sqrt{b_j} - \sqrt{a_j}\right)^2$$

$$D_P(\underline{a}II\underline{b}) = \sum_{j=1}^{n}\frac{\left(b_j - a_j\right)^2}{b_j}$$

$$D_F(\underline{a}II\underline{b}) = \sum_{j=1}^{n} b_j \log\frac{b_j}{a_j}$$

I. elv. Bridgman model (minimize average deviation)



Let the sum of the divergences among y and $\underline{t}_1,\dots\underline{t}_j,\dots\underline{t}_n$ vectors be minimal

Bridgman model 1.

- Our task is therefore to look for a mean vector y, for which the average deviation is
  minimum.
- $$\min \sum_{j=1}^{n} s_j D\left(\underline{y}II\underline{t}_j\right)$$
-
- *It is minimum if*

$$\frac{\partial D(\underline{y}II\underline{t})}{\partial y_i} = 0; \quad i = 1,\dots m$$

Bridgman model 1

$$\frac{\partial D(\underline{y}II\underline{t})}{\partial y} = \frac{\partial \sum_{j=1}^{n} s_j \left(y_i \ln\frac{y_i}{t_{ij}} - y_i + t_{ij}\right)}{\partial y_i} = \sum_{j=1}^{n} s_j\left(\ln y_i + \frac{y_i}{y_i} - \ln t_{ij} - 1\right)$$

$$= \sum_{j=1}^{n} s_j\left(\ln y_i - \ln t_{ij}\right) = 0$$

$$y_i = \exp\left(\sum_{j=1}^{n} s_j \ln(t_{ij})\right) = \prod_{j=1}^{n} t_{ij}^{\,s_j}, i = 1,\dots,m;$$

Bridgman model 2.

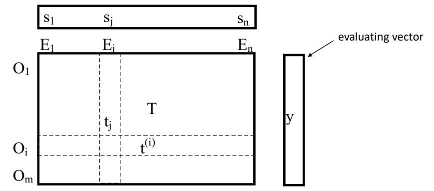
- Our task is therefore to look for a mean vector y, for which the average deviation is
- *minimum.*

$$\min \sum_{j=1}^{n} s_j D(\underline{t}_j II \underline{y})$$

- *It is minimum if*

$$y_i = \sum_{j=1}^{n} s_j t_{ij}, i = 1, \ldots, m$$

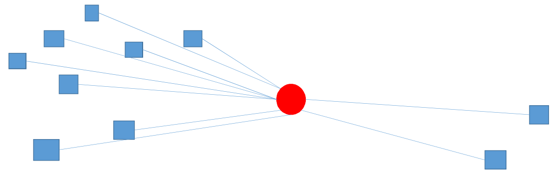
II. elv. Arimoto- Blahut model (minimizing maximum deviation)



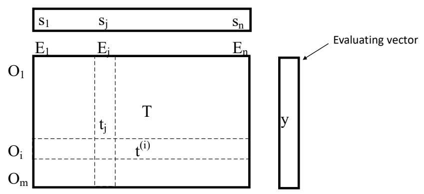
Let the biggest divergences among y and $\underline{t}_1, \ldots \underline{t}_j, \ldots \underline{t}_n$ vectors be minimal

Houses and the problem of the well

- Houses are given, where is the well?



III. Robust estimattion (minimizing median deviation)



Let median divergence among y $\underline{t}_1,\ldots\underline{t}_j,\ldots\underline{t}_n$ vectors be minimal

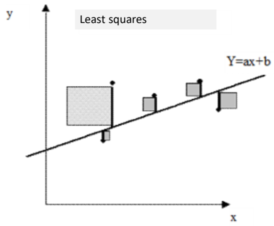
Principles for determining the evaluation vector

- Let's examine the deviation from the ideal $\underline{t}_1,\ldots\underline{t}_j,\ldots\underline{t}_n$
  Let minimum the sum of the difference between y and $\underline{t}_1,\ldots\underline{t}_j,\ldots\underline{t}_n$ vectors
  Let the largest difference between y and $\underline{t}_1,\ldots\underline{t}_j,\ldots\underline{t}_n$ vectors be minimal
  Let the median deviation between y and the $\underline{t}_1,\ldots\underline{t}_j,\ldots\underline{t}_n$ vectors be minimal

Mathematical problems:

$t_{i,j} > 0, \quad (i=1,\ldots,m)(j=1,\ldots n,)$	$t_{i,j} \geq 0, \quad (i=1,\ldots,m)(j=1,\ldots n,)$
Problem A1.: $\min \sum_{j=1}^{n} w_j D\left(\underline{y} II \underline{t}_j\right)$	Problem A2.: $\min \sum_{j=1}^{n} w_j D\left(\underline{t}_j II \underline{y}\right)$
Problem B1.: $\min_y \max_j D\left(\underline{y} II \underline{t}_j\right)$	Problem B2.: $\min_y \max_j D\left(\underline{t}_j II \underline{y}\right)$
Problem C1.: $\min_y med_j D\left(\underline{y} II \underline{t}_j\right)$	Problem C2.: $\min_y med_j D\left(\underline{t}_j II \underline{y}\right)$

Why median? a robust estimation



example

$$\sum_{j=1}^{n}(y_j - f(x_j))^2 \to \min$$

Y=2x+3

X	2	3	4	5	6
Y	7	9	11	13	15

Y=30x-81

X	2	3	4	5	6
Y	7	9	11	13	155

Example cont.

$$\underset{j}{med}(y_j - ax_j - b)^2 \to \min$$

Y=2x+3

X	2	3	4	5	6
Y	7	9	11	13	15

Y=2x+3

X	2	3	4	5	6
Y	7	9	11	13	155

Example cont.

$$\sum_{j=1}^{n}\left(y_j - f(x_j)\right)^2 \rightarrow \min \qquad \underset{j}{med}\left(y_j - ax_j - b\right)^2 \rightarrow \min$$

X	2	3	4	5	6
Y	7	9	11	133	155

Y=42X-105

Y=2X+3