RELEASING RESTRAINTS ON NETWORK SCHEDULING TECHNIQUES

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Abstract

In 1989 engineers of Hungarian Railway Company (MÁV) asked experts of Department of Construction Technology and Management (DCT&M) of Budapest University of Technology and Economics (BUTE) to develop a Computer Aided Decision Supporting system to assist scheduling periodic and by-need reconstructional and maintenance works along the whole railway system. Challenge of modelling the problem was the Permanent Scheduling Job of works on a 3-years long rolling basis, including thousands of jobs within accuracy of minutes (due to ongoing operation). Answering key questions of elaboration enforced researchers to revise capabilities of well-known network-typed scheduling techniques (CPM, PERT, MPM, PDM) trying to release restarints on their application. Results have been referred and taught since then as General Time Model (GTM) based on a version of Floyd–Warshall algorithm. The paper introduces key ideas of new considerations.

Key words

operations research, graphs, network techniques, computer applications

1 INTRODUCTION

In early applications (CPM [4], PERT [5]) graph structures for scheduling had been restricted to a very strictly restricted topology addressed as network. Namely: a directed weighted connected graph with one starting node (origin), with one ending node (terminal node), with no loops and with no negative weights on the edges. Necessity of these restrictions on graph structure can be ascribed mainly to early solution algorithms, such as labelling techniques, and to capabilities of early computers the applications had been run on.

It can be shown that without the rest of before mentioned restrictions, on general directed weighted graphs, valid and calculable time models can be interpreted for use of any level of project and/or production management. Moreover, in widely known MPM [7,8] and PDM [5,13] techniques loops and negative weights have been implicitly integrated in the models resulting in no any unexpected, contrary and/or unsolvable conditions. Furthermore the practice of originating all initial steps/tasks from one single starting node in the model, and/or directing all finishing procedures/tasks into one single ending node may integrate unintended

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and misguiding information, when modelling and analyzing a complex project under examination.

2 THE NEED

The need for revising restrictions of „traditional“ network techniques emerged at a research project while elaborating a Computer Aided Supporting System assisting managerial decisions at planning and controlling under-operation reconstructional and maintenance works of hungarian railways' system [9].

Challenge of management problem was the „Permanent Scheduling” of works on a 3-years slipping time-span looking over thousands of jobs with accuracy of minutes. No expressed start, no expressed end, widely diversed responsibilities, dispersed locations across the country, but one complex „must-be-operating” (under traffic) railway system and a restricted common pool of some significant specialized resource series.

Traditional Scheduling techniques (including traditional Network Techniques) proved to be insufficient. „The project to be scheduled was not a project.” Conclusions of revisional studies on network-techniques have been discussed as „General Time Model“ (GTM) in curricula of elective subjects of Department of Construction Technology and Management of Budapest University of Technology and Management since nineties of last Century.

3 MATHEMATICS BEHIND

The scheduling problem, itself, with pre-set durations of tasks (weights) and with pre-set precedence relations amongst them (edges) can be derived from a primal-dual problem-couple widely known in operations research, namely: the longest path problem on a weighted graph (Kelly-Walker, CPM), and the minimum potential's problem with lower bounds on differences of pairs of potentials (Roy, MPM).

Exposed or not, usual algorithms (manual or automated) developed for to solve the scheduling problem are focusing on the minimum potential's problem meanwhile executing a kind of implicit labelling technique [1,3,6,13]. (Calculations are started at the origin and are rolling towards the terminal node, then back – “forward pass” and “backward pass”).

Setting The Longest Path Problem in the focus of examinations necessity of preset restrictions can be reduced radically. Length of any path – though the term itself is directed, you have to state where from where to – on a weighted graph is interpreted as pure sum of weights of edges constituting the given path. And, it is irrelevant, from the point of view of the result, in what a sequence the members (weights of edges) are added together. Thus, no need for pre-set origin, pre-set terminal node, neither numbers of them are important during the calculations, length and edges of longest paths can be identified, and knowing the longest paths time potentials (early and late times) can be assigned along them to the events and to activities represented by nodes and/or edges of the graph model.

In case of a CPM/PERT time model considering the logic of calculation and the linkage between the Longest Path Problem and The Minimum Potentials' Problem calculations can be summarized as:

$\Pi_i \geq 0; \quad \forall i \quad i \in N$ (*non-negative potentials should be assigned to all nodes of the graph*)

$\Pi_j - \Pi_i \geq \tau_{ij} \quad \forall ij \;\; ij \in E$ — (*differences of pairs of potentials are limited by lower bounds, testified by directed weighted edges of the graph, where lower bound values are the weights of the edges*)

$\Pi_{max} \rightarrow \min$ — (*the largest value in the established potential system would be at minimum, which equals to the length of the longest path from the origin to the terminal node*)

(*Where N is set of nodes (i), E is set of edges (ij), τ_{ij} is the lower bound value that is the weight at edge ij, Π_i is the time potential to be assigned to the node i*)

In case of a MPM/PDM time model tasks (activities/processes) having pre-set fixed durations are represented by nodes of the graph, hiding significant characteristics, that on any path, including the longest one, a node has a significant extent (its duration).

Due to elementary rules of algebra, multiplying inequality representing bound on difference of a pair of potentials by minus one, any upper bound can be equally substituted by a lower bound (reversing the direction of subtraction, that is direction of edge, and changing the sign of the limit value) Thus, a mixed bounding system can be transformed to a homogeneous one (having “lower” bounds only).

$\Pi_j - \Pi_i \leq \tau_{ij} \quad / \cdot(-1)$ — (*upper bound*) (1)

$\Pi_i - \Pi_j \geq -\tau_{ij}$ — (*upper bound turned to lower bound*) (2)

Analogically, any fixed duration of a task can be set by a pair of lower and of upper bounds having the same limit values (τ_{SF}, that is its duration) between its start (S) and its finish (F).

$(\Pi_F - \Pi_S = \tau_{SF}) \quad \equiv \quad (\Pi_F - \Pi_S \geq \tau_{SF}) \;\cup\; (\Pi_S - \Pi_F \geq -\tau_{SF})$ (3)

As a consequence of above, loop of directed edges is given (between the starting and finishing “nodes” of the task), negative weight is given (upper bounding for fixed duration), while analogy of the Longest Path Problem is not damaged and the model is also calculable.

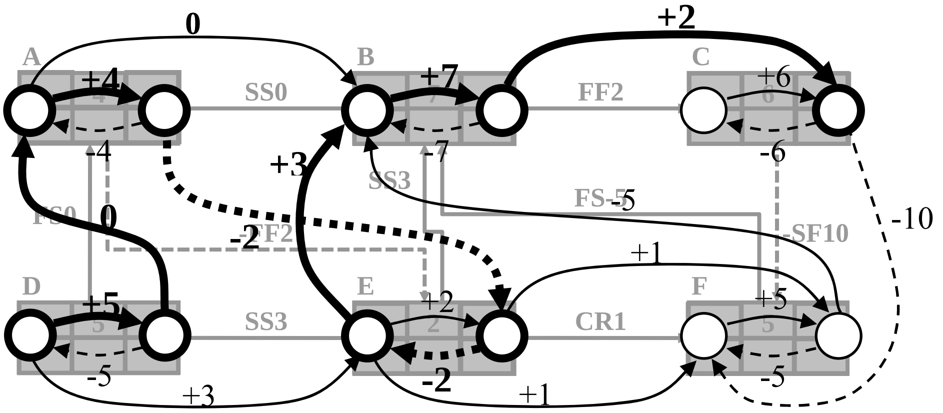

Fig. 1) A “radiographic view” (standard DiGraph representation) of a typical MPM/PDM network with fixed durations (boxes and arrows in gray in the background), with loops, positive (lower bounds - solid arrows) and negative (upper bounds - broken arrows) weights. (From Syllabi of BUTE-DCT&M; www.ekt.bme.hu)

The only problem may emerge if contradictious conditions (any loop with positive length, that is where sum of weights of edges forming the loop is greater than zero) is set in the model. Though it should be mentioned that researches are going on to study the behaviour of positive loops in time models, generating “endless”, periodically repeated jobs in production models.

An interesting, not widely known phenomenon as a consequence of negative weights inserted is the so called paradox situation, when decreasing the duration (bounds, in absolute value) of a task results in an increment in the length of the longest path (increases the overall time of the project). Such a situation may occur when edges with negative weights are included in the longest path. (Consider MPM task “E” in Fig.1.)

To calculate network models with negative weights and with loops, traditional rolling typed calculations (such as labelling techniques) may prove inefficient, and may necessitate some kind of iterative approach and/or overwhelming analysis of all possible solutions. The earlier can be based on recognition that by each iteration at least one potential (Π_i) gains its final value, while for the later either a modified Floyd-Warshall algorithm can be applied [2,11].

4 RESTRAINTS TO RELEASE

Seeing no mathematical difficulties or contradictions when releasing early restrictions (no loops, no negative weights, etc.) on a network time model let’s review restraints pressed on the user of traditional network techniques from the point of view of construction practice.

4.1 With no loops and with no negative weights

Negative weights (upper bounds) in a dynamic time model are of much use when modelling technological processes. One should think of the so called sensitive conditions (supporting the earth-wall of the trench after excavation; blinding after refinery excavation for to preserve soil conditions; demolishing upper segment of a soil-mech pile for interworking its reinforcement with that of pile-cap; etc.) and of resource management aspects (bounding and/or excluding idle times).

Even more – as it is in reality – durations of tasks (activities) are usually bounded both from up (economic aspects) and from down (technical aspects), and the applicable or “optimal” duration can be resulted from time analysis of the model itself (exact values of durations of tasks can be output of network calculations, while real input is the bounding system only).

There is no mathematical need for to set exact task (activity) durations in advance.

Double limitation (lower bound and upper bound together) ab ovo vindicate arrow loops when representing them graphically as introduced above (See Fig.1.).

4.2 Connected graph, with one starting node and with one ending node

In case of complex and/or regional development project contractors of various industries are co-operating for to achieve the aim set for the project. A project can (even more must) have a well defined initiation (launch), and a well defined target (delivery, state, etc.). But nearly all the contributors do have their own interests and their management preferences out of the common project too. For them the project is not a project – in classical sense of a project – but one (bigger or smaller) item in their business (production) management.

Do consider a hydro-electric power station. Massive works of landscaping (dams, access roads, reservoir, etc.), sub- and superstructure construction (turbine shafts, generator house, operator centre, etc.) and manufacturing and installing electro-technical equipments (turbines, generators, transmission lines, switch stations, etc.) are to be executed. It is hard to believe that a float (slack) indicated by a closed network (one origin, one terminal node) due to time-consuming preliminary earthworks can be utilized during manufacturing electro-technical equipments. (The manufacturer is unwilling to assign its resources to the project immediately from the launch of the project, if it is not necessary. It would produce for other consumers instead, and would have its "own" starting point for contributing the project in proper time.)

Anyway, as pointed above, for algorithms focusing on the Longest Path Problem there is no need to have one single origin and one single terminal node. Besides, any float (slack) is manageable if necessary resources are dedicated to the project exclusively.

As a consequence, it is not needed that the model (graph) be connected. Independent, parallel projects can be studied simultaneously, and can be analysed from view point of other – say environmental and/or sustainability – consequences. (See: "multi-project management").

4.3 Directed graph

Probably the most staggering recognition: sometimes, the graph developed for modelling relationships in time do not need to be directed! One should think of finishing jobs of building a dwelling house, when succeeding crafts are sharing the same limited location (room, surface, etc.), but they have no specific technological relations to each other (say: installing electric fittings in the walls and on the ceiling, and laying tiles on the floor), so their sequence is occasional or is set according to other (non technological) aspects.

An interesting modelling challenge can be scheduling of construction of a large waterproof foundation slab (or basin) to be concreted in special pattern of blocks, letting shrinkage of concrete happen before neighbouring units get be casted. Restrictions are on minimum ages of blocks, but not on sequence of them. (Undirected graph edges can indicate adjacencies of neighbouring blocks, while weights can represent expected differences of ages.)

Ruefully, for to calculate a model of this kind, Combinatorics is holding a great barrier, and usually some kind of Branch & Bound technique or Heuristics would be applied, running ("solution") time of which is depending mainly on the data themselves. [1,3,6,10]

5 CONCLUSION

Applied Mathematics (Operations Research, Management Science) had developed much since the boom of fifties and sixties of the last century. Modern computer technology is besieging frontiers of material world. Though modern and high-capacity tools for aiding solutions of practical problems are available, Construction Management seems to be contented with some smart old modelling techniques had been developed in early years of Management Science.

Pointing out unnecessary and obsolete (mathematically undue) restraints set on network time models, the same time feeling would-be utility (and lack) of releasing restraints mentioned above, "exclusive" use (and education) of traditional network techniques is going to be less acceptable. The contradiction is even more evident when considering Construction in its production (Production Management) environment.

While respecting and paying honour to developers of well known and approved “traditional” network techniques (CPM, PERT, MPM, PDM) we have to face the fact that their terms, interpretations, restrictions, solution algorithms even their inputs and outputs got be worth rethinking. Traditional terms as “start”, “termination”, “critical path”, “loop” (see Fig.1.) are worth rejudging. Characteristics earlier reckoned as obstacles and caused difficulties at handling time-modelling techniques can be rehabilitated and can be of much more use then some would think it.

A general time model (GTM), initiated by a serious practical problem, had been elaborated at BUTE-DCT&M, without above mentioned restrictions, and also without harming analogies of problem-couple of The Longest Path Problem and The Minimum Potentials Problem. Essence and key ideas of it have been integrated in curricula of students of Civil Engineering studies.

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