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Photos by Andy Ryan

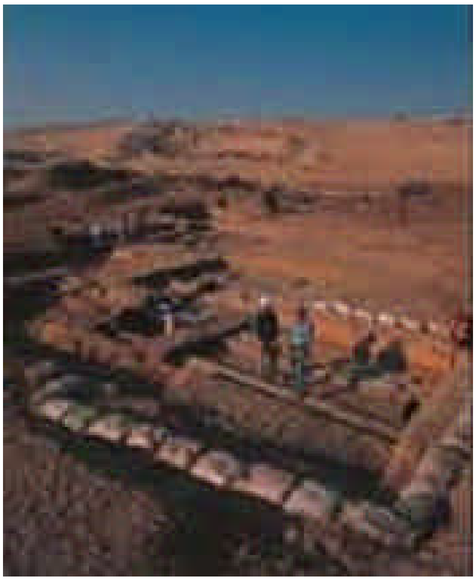
Craig B. Smith, P.E.

A process of forensic analysis that applied modern-day technology to bridge the chasm of time provides some surprising answers to the question of how the Great Pyramid at Giza was built.

The construction of the Great Pyramid at Giza is one of the marvels of the ancient world. Originally 481 ft (147 m) high—the top 30 ft (9 m) have been lost to the ravages of time—the pyramid rests on a base that covers an area of 13.1 acres (5.3 ha), incorporates 3.4 million cu yd (2.6 million m3) of material, and is roughly two-thirds the size of Hoover Dam. For centuries mankind has wondered how the early Egyptians were able to accurately level the site, position enormous blocks of limestone and granite—some weighing as much as 20 tons (18 Mg)—and then construct the immense structure with great precision in terms of both its dimensions and orientation.

In addition to the construction challenges the project posed, it required a sophisticated approach to program and construction management. The project entailed the staging of a remarkable construction undertaking that required the marshaling of vast amounts of materials from all over the Egyptian kingdom; the feeding, housing, and payment of thousands of workers; and the scheduling of the work for timely completion—that is, prior to the death of the pharaoh.

Working closely with leading Egyptologists in both Egypt and the United States—most notably, Mark Lehner, an Egyptologist with the Harvard Semitic Museum—a team of construction managers with the international architectural, engineering, and construction management firm Daniel, Mann, Johnson, & Mendenhall (DMJM) performed a forensic analysis to determine the construction methods and construction management techniques that were employed by the ancient Egyptians in constructing the Great Pyramid. By applying modern program management and construction management methods the project team developed a number of interesting insights into centuries-old mysteries concerning the size of the workforce, the duration of construction, and the design of the construction ramp.



Khufu's pyramid, which is often referred to as the Great Pyramid, is a structural marvel. The remains of an ancient construction ramp are visible in the foreground. Craig Smith (at left) and noted Egyptologist Mark Lehner stand in the remains of the workers' village, which Lehner excavated.

Program management is the science and practice of managing large private and public projects. DMJM functions as the program manager for projects around the globe, managing large, complex programs for clients in both the public and private sectors. The logistical issues—making certain everything comes together at the right time, in the right quantity, with the right quality—are among the greatest challenges of these projects and become the major preoccupation of the program manager. To clearly illustrate the complex activities undertaken by a program manager in today's environment, DMJM sought a compelling example that would be familiar to most people. Someone commented, "If you think managing today's projects is complex, try building the Great Pyramid!" And thus, our project—Program/Construction Management in 2550 b.c.: Building the Great Pyramid at Giza—was born.

Initially, our goal was simply to identify the major steps that a hypothetical program manager would have undertaken to construct the Great Pyramid at Giza. We asked the team of construction managers to visualize the work that would be required so that we could prepare logic diagrams, schedules, and other tools of the program manager. But as the project unfolded a strange transformation took place: Members of the team became absorbed by the challenge. How would you build the Great Pyramid?

Engineering, mathematics, and science—disciplines necessary to execute large construction projects—were well established in ancient Egypt. The Egyptians could predict the flooding of the Nile, identify major stars and the position of the stellar bodies with some accuracy, and calculate areas and volumes of structures as complex as the pyramids. In addition to having a system of written records, they used many basic tools made of copper, including saws, chisels, hammers, and drills, and understood the principles of the lever and the inclined ramp. It is reasonable to assume, then, that they possessed both the ability and the resources to undertake a project as complex as the construction of the Great Pyramid at Giza.

Also known as the Great Pyramid of Khufu—Khufu reigned from 2551 to 2528 b.c.—it was constructed during the fourth dynasty, about 2,550 years before the birth of Christ, and is the best known and largest of the 80 pyramids discovered along the west bank of the Nile. Indeed, for more than 4,000 years it was the largest man-made structure in the world.

The logistics involved in the construction of this pyramid are staggering when one considers that the ancient Egyptians had no pulleys, no wheels, and no iron tools. Large blocks of limestone and granite—some weighing as much as 20 tons (18 Mg)—had to be cut at quarries and transported by boat across or down the Nile River. All of the interior rock was carved on the Giza plateau, but the limestone used on the exterior facing of the pyramid came from Tura, which was situated across the Nile. Blocks of limestone weighing anywhere from 2.5 to 6 tons (2.3 to 5.4 Mg) made up the bulk of the structure. Estimates indicate that more than 2 million such blocks were used. Most of these were cut from a quarry at Giza; heavier blocks of granite from Aswan were used to construct the King/s Chamber.

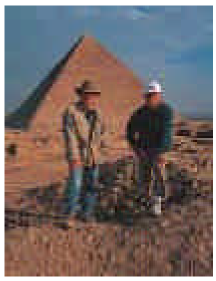
Egyptian workmen perfected the technique of cutting holes in stone faces with hand-driven drills. Wedges were then inserted into the holes, and slabs of stone were broken loose by pounding on the wedges with mallets. The slabs were subsequently dressed down to finished dimensions.

The final dimensions of these stones were extremely accurate on the exterior faces of the pyramid: the joints were made within fractions of an inch—n some cases substantially less than 1/8 in. (3 mm). The pyramid was oriented with its major sides either north-south or east-west. This in itself was a remarkable undertaking, given the accuracy to which it was done, because the Egyptians had to perform the work using astronomical or solar observations—the compass had not yet been invented. The dimensions of the pyramid are extremely accurate and the site was leveled within a fraction of an inch over the entire base. This is comparable to the accuracy possible with modern construction methods and laser leveling.

The Greek historian Herodotus wrote that the construction of the ramp and pyramid occupied 30 years with a workforce of 100,000 men. There is also speculation that some of the workforce was seasonal, consisting largely of farmers who arrived during the periods when the Nile flooded and they were unable to work in their fields. The 100,000 figure seems high in the light of what we know today, but by any standard and from any point of view this was a mammoth undertaking.

Excavations indicate the presence of an artisans' village, which may have housed some 4,000 to 5,000 people. This, plus evidence of tools and workshops, led us to surmise that there was a full-time workforce of about 4,000 to 5,000, not including the workers responsible for cutting limestone at the distant quarries, transporting it to Giza, and bringing it to the site. This number also did not include the administrative and support staff necessary to feed and care for a permanent workforce or those necessary to handle the logistics of bringing in supplies, timbers for scaffolding and rollers, stone blocks, and other construction materials.

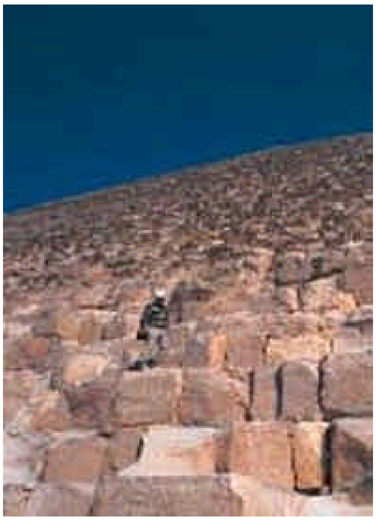
Craig Smith and Mark Lehner worked closely on the forensic analysis that explained how the Great Pyramid was built. The photograph left, which is a view of the Queen's Pyramids at the Pyramid of Khufu, indicated how surface stones were finished.



Several theories have been advanced as to how the pyramid was actually built. Herodotus indicated that a system of levers was used. Long wooden poles were employed to elevate the blocks from one level of the pyramid terrace to the next level. Either multiple levers were used or the levers themselves were moved to each elevation as it became necessary to lift higher and higher. We determined that this approach would have been impractical. There is considerable evidence, however, to support a different approach: that of an inclined ramp.

We know that sloping ramps were constructed for other pyramids; thus we speculated that after the site was leveled, an initial course of blocks was placed to outline the base of the pyramid. This placement was done with extreme care because it formed a reference point for the other dimensions as construction proceeded. We also surmised that in the center of the area encompassed by the base of the pyramid a rock outcropping remained to some unknown height and area. Construction continued for a number of courses, although a tunnel, called the descending corridor, was cut into the base of the pyramid. This corridor descended a distance of some 350 ft (107 m)—roughly to the center of the pyramid and beneath it—angling down at about 26 degrees, where a room was constructed.

The unfinished state of this room indicates that the plans changed. An ascending corridor was constructed from the descending corridor, rising to the center of the lower portion of the pyramid. Here another chamber was built, which is known today as the Queen's Chamber. Above the Queen's Chamber is a large, lengthy, and unique gallery—the Grand Gallery—which leads to the King's Chamber. This was the intended final resting spot of King Khufu.



The size of the blocks used in the lower courses of the Pyramid of Khufu is evident in this view.

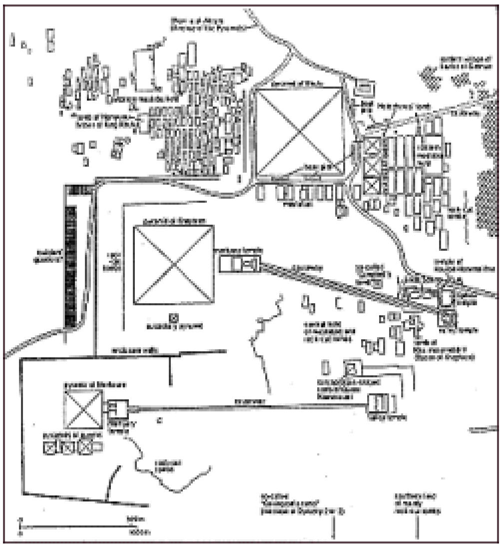
As construction reached the level of the corridor and chamber it was possible for workmen to install the highly finished walls, lintels, and ceiling blocks of the corridors and chambers from a level surface and then build the rest of the pyramid up around them as they proceeded upward. We surmise that the ramp was extended until the top portion of the pyramid was constructed, and at some point a limestone capstone was put in place.

The exterior courses of the pyramid were constructed of white Tura limestone casing stones, which were cut and fitted more accurately than the interior core stones. These stones were placed with excess material remaining on the face, to be trimmed at a later time. Once the last row of casing stones was in place, the ramp and scaffolding were removed to expose several courses of the casing stones. Then the outer surface was trimmed to the finish dimensions to give the pyramid a smooth exterior surface. This was done working downward, as the ramp and scaffolding were removed.

Team members drew upon their expertise in working on large construction projects to determine how long it would take to cut and transport blocks of limestone and to erect the pyramid. They tried to imagine the time that would be required without the availability of modern tools. The first step in this process was to develop a work breakdown structure—that is, to define the various elements of the work to be performed.

Once we had the work breakdown structure, we developed a logic diagram to illustrate the construction sequence that the team found most plausible. Construction estimators researched the methods used before the introduction of machinery in order to produce labor estimates for each of the tasks identified. Where possible these estimates were checked against published data, but for the most part the team relied on the experience of its members. Collectively, the team members have overseen large construction projects in Saudi Arabia involving tens of thousands of workers. Additionally, several members had experience with labor-intensive construction methods in Third World countries, including the processes of hand excavation and the pouring of cement by the bucketful.

These unit estimates were then combined with the engineering estimates of materials and construction methodology to derive the amount of time and manpower required to perform each element of the work. The data obtained were then used to develop a critical path construction schedule. The analyses enabled the team to reject as impossible certain hypothesized construction methods. For example, a single ramp to the top of the pyramid would have extended for more than half a mile and would have involved more construction than the pyramid itself. Likewise, ramps to each face were found to be unnecessary.



The Giza site plan indicates the locations of three primary pyramids situated here: the Pyramid of Khufu—the Great Pryamid; the Pyramid of Khephren; and the Pyramid of Menkaure.

We determined, however, that some type of ramp structure was probably used given the remains of ramps at other sites and our assessment of available construction methods. A single large ramp to level 50 of the pyramid would have been of reasonable height and volume; it would have permitted two-thirds of the blocks to be put in place. The team postulated that after level 50 a square helical ramp would have been constructed on the pyramid itself to reach the upper layers. At this point the number of blocks decreases and constraints on block delivery are not as restrictive. At the apex of the pyramid—the last 10 to 20 levels—the number of blocks is very small. The team suggested that an internal "staircase" was created and that levers were used to place the capstone and the last remaining blocks.

The critical path analysis showed that the production of blocks from the quarry would not have been a constraint. Additionally, we determined that blocks could have been prepared in advance and stockpiled on-site in the event of a shortage of stonecutters. We assumed a large number of workmen could be recruited on a seasonal basis to assist in transporting the blocks up the ramps to the working area, where skilled masons put them in place and built the corridors and chambers.

The team also worked out the logistics for site preparation, quarry operations, transportation of the finished limestone from Tura and granite from Aswan, the creation of a workers' village for permanent skilled staff, construction of the ramps, performance of the finish work, and removal of the ramps at the end of construction.

Based on our program management approach and our informed guesses we concluded that the total project required an average workforce of 13,200 persons and a peak workforce of 40,000 and that it required two to three years of site preparation, five years of pyramid construction, and two years of ramp removal, decoration, and other ancillary tasks. Assembling a workforce of this size—and feeding it—appear to have been well within the capabilities of the Egyptian economy at that time if the population was in fact 1 million to 1.5 million.

No records have been found that relate to the design of the Great Pyramid. However, drawings have been discovered for tombs constructed during later dynasties. Additionally, plans and other records exist indicating that at the time of Khufu's reign the Egyptians knew how to calculate volumes, areas, and angles; that they knew how to level a site and construct right angles; that they could survey and use solar and astronomical observations to orient structures; that they understood the concepts of structural load transfer and apparently had an idea of the load-bearing capacity of the materials they used; and that they recognized the need for an adequate foundation for the structure. Because they could calculate the volume of ramps, we assume they optimized the planning of the construction to minimize the materials and labor required just as we would today so that labor would be expended on the most critical and challenging aspects of the project. We speculated that the pyramid's design was actually implemented by a master builder or overseer who had worked on another pyramid.

Excavations at Giza show old quarry marks and the remains of a workers' village. Later excavations at Deir el Medina, where extensive records have been recovered, support the theory that there was a permanent labor encampment at the site that housed the skilled stonemasons, draftsmen, and overseers. The village no doubt had the capacity to support the workforce, because there is evidence of a bakery—and even tombs for those who died during the course of the construction. Other aspects of site preparation we considered were the construction of housing, sanitary facilities, workshops, roads from the quarry at Giza to the pyramid site, and docks. (The time of the annual Nile flooding would have been the ideal time to bring in goods by ship because the floodwaters came within a quarter of a mile of the Giza site.)

We speculate that the pyramid site was surveyed and then excavated to bedrock, which would have provided a firm foundation for the pyramid structure. The Giza plateau was no doubt selected as the site for the pyramid because of the available limestone and the site's proximity to the pharaoh's residence. Upon removing the loose material the builders left a rock ridge that was later incorporated into the pyramid structure. To avoid handling material any more than was necessary, it is likely that the cut material was moved to the location of the construction ramp and placed so it could be part of the ramp.

Once bedrock was exposed the site was leveled. This was most likely done by the use of a square level—a right angle with a cross piece resembling the letter A with a plumb bob that hung from the apex and registered against the cross piece. Leveling was done in a series of measurements that established benchmarks along the length of the foundation.

Another theory held that leveling was done by constructing a series of mud canals across the site, filling them with water, and measuring the depth from the water surface to the rock beneath (establishing baseline measurements and survey points). However, we discarded this idea because of the effort that would have been involved in hauling water to cover such a volume of canals and the losses that would have resulted from evaporation and leakage.

Next, using either solar observations or star sightings, survey working points were established and corner positions were fixed. Since the Egyptians worshiped the sun, it is more credible to us that they understood the movement of the sun and would have measured the sun's shadow to determine true north. In a simple experiment with the tools and knowledge available to the ancient Egyptians, we found that this determination can readily be made.

At this point tunneling to construct the descending corridor and lower chamber was probably started. A construction gap was left open in the core blocks while the descending and ascending corridors, the Grand Gallery, and the King's Chamber were constructed.

In this view of the terrace at the Pyramid of Khufu, square "plugs," believed to mark an ancient surveryor's measurement posts, are clearly visible.

Quarry operations at Giza began concurrently with the commencement of site preparation. We assume that the bulk of construction material came from the area of the site to minimize transport of heavy blocks. Archaeological evidence supports the contention that there were one or more quarries on-site. We assumed a workforce of sufficient size to keep up with the rate of block installation. (A smaller force could have been used if work started a year in advance to build up a stockpile of blocks.)

The first step in construction would have been to lay the ground course. This process would have consisted in placing large blocks with great precision to establish the dimensions of the pyramid. Based on a survey reported in the literature, the base is square and is oriented to the four points of the compass to standards that would be challenging to a builder today. Construction would have proceeded to add layers above the base, until the next "step" was achieved. Here the structure would have been carefully leveled again. It would not have been necessary to level each layer, as this would have increased the amount of cutting and trimming on each block and would have wasted material. It is believed that there are 14 to 16 layers per step and 15 to 17 steps.

We assumed that construction was based on three components: the outer casing stones—carefully dressed white Tura limestone; an inner layer of "backing stones"; and the core blocks of Giza limestone, which were not dressed accurately but were fitted into the inner volume of the pyramid and then leveled only at the next step (not every course). Irregular shapes were incorporated into the structure to maximize the use of available materials. Casing blocks would be field dressed so as to fit accurately next to and on top of their adjoining blocks.

We considered many concepts to understand how the Egyptians were able to raise blocks to a height of 481 ft (147 m) with the limited tools available. We assumed the use of rollers but not wheels or pulleys. To evaluate the ramp issue we first constructed several mathematical models that computed the number of blocks per layer and the volume, height, and other measurements of the blocks. We know the blocks are not of uniform dimension—that the lower blocks are thicker by as much as 5 ft (1.5 m) while the thickness drops to 2 ft (0.6 m) or less near the top. Not having a survey of typical sizes, we made a series of calculations based on average sizes (see illustration).

Our calculations convinced us that most of the ramp concepts would have been impractical because they involved a construction effort greater than that required for the pyramid itself. We assumed that the Egyptians would not commit resources to building anything more than minimally required given the fact that the ramp had to be demolished at the conclusion of construction.

The literature reports that the Great Pyramid is constructed of 2.3 million blocks and that each weighs on average 2.5 tons (2.3 Mg). Our review found no basis or origin for these numbers, which have been widely quoted. We made our own estimates, assuming various dimensions and a specific weight for limestone of 160 lb/cu ft (2,563 kg/m3). These calculations showed that there could be from 2 million to 2.8 million blocks, depending on the assumptions. We then refined the calculations to deduct for the void volume of corridors and chambers, subtracted an allowance for granite used in lintels, the capstone, and ceilings, and treated the finish layer separately. This suggested that the basic building blocks numbered about 2 million, based on average dimensions of 3 ft (0.9 m) wide, 3.5 ft (1 m) high, and 4 ft (1.2 m) long.

An interesting possibility is that the capstone might have been brought up to the last level that was reachable by a ramp and then jacked up as the balance of the pyramid was constructed—that is , the pyramid was built beneath it and it rose with the remaining levels.

Inspection of our mathematical model showed that at the point that layer or level 50 had been reached essentially two-thirds of the blocks had been put in place. This suggests that a single large ramp—to one face of the pyramid—would have been feasible. This ramp would have been 175 ft (23 m) high and more than 1,000 ft (705 m) long and would have had a grade of 15 percent—which we assumed as an upper limit. Also, it would have contained 30 percent of the volume of the pyramid itself. The ramp dimensions would have been influenced by the construction schedule. To construct the pyramid on a reasonable schedule the ramp would have had to be wide enough to enable multiple teams to approach the working surface, deliver their loads, and leave without hindering other workers.

Ultimately we settled on a hybrid ramp scheme. There was a single ramp on one face of the pyramid up to level 50, from which a series of ramps wrapping around the pyramid reached level 120. These ramps would have been much narrower and supported by the pyramid itself and thus could have been constructed with much less material. We hypothesized that the blocks in the last two (outer) courses were left out near the corner to create a takeoff point wide enough for the primary ramp. The secondary ramps would have been used at an elevation at which the horizontal distance was long enough for a significant gain in elevation.

We assumed that a third method was used above this point: the "staircase" left in the center of the construction at the very top. The blocks for the peak would have been pushed manually from below and pulled up by ropes over poles or bearing stones up this staircase and then put in place. At this point the number of blocks required is only about 7,000 for the last 20 layers. Once the capstone had been maneuvered into place the staircase would have been filled in from the top down to the platform level at the end of the last ramp.

An interesting possibility to consider is that the capstone might have been brought up to the last level that was reachable by a ramp and then jacked up as the balance of the pyramid was constructed—that is, the pyramid was built beneath it and it rose with the remaining levels.

The pyramid was finished with white limestone casing stones from the quarry across the Nile River at Tura. We assume that the finish blocks were brought by ship to Giza. These blocks were carefully placed, then trimmed after placement to provide a smooth exterior surface. Using the same model to calculate the number of finish blocks that we had used to determine the number of blocks per layer, we determined that the number was approximately 53,000.

We assumed that scaffolding was erected at the top levels to position these blocks and that the work proceeded upward course by course. Because the topmost blocks were half the size of the regular blocks, they could be positioned by hand. Once the work reached the top of the pyramid any missing blocks were filled in down the staircase and any finishing touches were performed. As layers were completed the ramp was gradually removed.

We determined that there were 3 workweeks of 10 days per month—8 days of work followed by 1 to 2 days off. A workday consisted of four to five hours in the morning followed by four to five hours after lunch. Deductions would be necessary for holidays and religious observances, so we used 280 working days per year as our estimate for construction time.

We estimated that a delivery rate of 180 blocks per hour was required from level 50 to level 74 and then used this rate to determine if the ramp size and number of crews were feasible. This seemed possible. We then determined that at the lower level the ramp would be wider and could sustain delivery rates twice this number. Above level 75 the delivery rate drops off because of the smaller number of blocks, so ramp size and crew numbers are reduced. The size of crews can be estimated in various ways. Carrying capacity will ultimately depend on load and distance. We assumed an average crew of 20 men.

This detail indicates how casing stones were cut to the proper angle.

Unit cost estimates were developed from a variety of sources, including the team's judgment and experience. For example, our stonecutting estimate of two man-days per block is based on our judgment. For the average block we assumed that a team of 20 laborers was required to pull a sled up the ramp and onto the work area. This would require four hours on average (up to level 50), which meant that a team could move two blocks per day. Ten man-days were required, therefore, to move each block into place.

For estimations regarding excavation and ramp construction, we consulted turn-of-the-century civil engineering handbooks and established unit rates for moving earth manually. This corresponded to about 1 cu yd/h (0.8 m3/h), with time added depending on the distance the material was carried. We estimated that at an average distance the rate was 0.03 d/cu ft (0.1 d/m3). We also prepared a manpower labor forecast. Once courses 1 through 50 were completed the labor requirements dropped off considerably; additionally, the skilled labor requirements are consistent with a workers' village of 4,000 to 5,000 persons on-site. The total labor expended is 36.7 million days, or approximately 131,200 man-years. Thus the average labor force over the 10-year duration of the project is therefore 13,200 men.

We learned that workers were paid in grain—to make bread and beer—as well as in oil, other foods, and cloth. Payments differed, of course, depending on the level of skill and rank. Ancient records indicate that a superintendent earned 8 jugs of beer and 16 loaves of bread daily. We arbitrarily prorated these numbers to estimate payments to other classes of workers. While this is undoubtedly an oversimplification, it provides a rough measure of the total cost of the construction. There was a barter economy in place then, so a worker with one set of skills might perform work for another, who would return the favor by making something for him. There was also some moonlighting going on as workers used their free time to work for third parties. Thus the total labor costs for construction of the pyramid were approximately 111 million jugs of beer and 126 million loaves of bread over the 10-year span of the project. The production capacity for agrarian Egypt at that time suggests that it was perfectly plausible for the economy to support such an undertaking over that period of time.

While there is uncertainty as to precisely how the Egyptians built the Great Pyramid, there is certainty about the fact that it was done. The pyramid stands today as awesome testimony to the skill and sheer determination of the ancient race that built it. We must also stand in awe of their program management techniques, as it is equally certain that they had highly developed administrative and planning skills. The complexity and logistical requirements of this project are simply extraordinary.

Craig B. Smith, P.E., Ph.D., is the chief operating officer of Daniel, Mann, Johnson & Mendenhall in Los Angeles. He explained this project in the television special "The Great Builders of Egypt," which aired on the Arts & Entertainment channel earlier this year.

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