## Construction Management

# NETWORK TECHNIQUES

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## Koenigsberg, Prussia, 18<sup>th</sup> Century

now: Kaliningrag, Russia



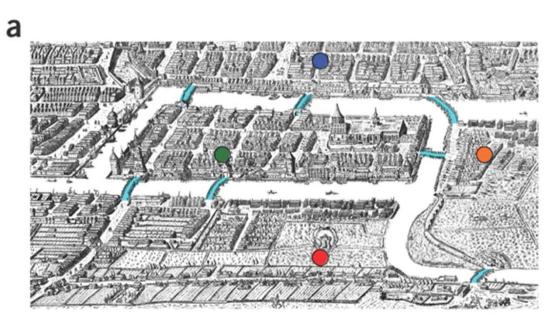
# Problem: Is it possible to take a round trip through the city crossing each of its seven bridges once and only once?



#### "Seven Bridges of Koenigsberg"

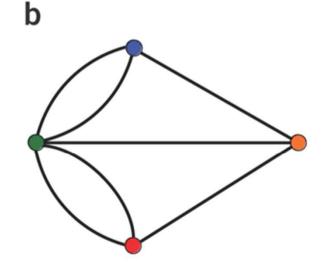
Problem: Is it possible to take a round trip through the city crossing each of its seven bridges once and only once?

Solution and Proof: Leonhard Euler, 1735 Basics of Graph Theory





Leonhard Euler (1707-1783)



### G R A P H

#### as a model structure:

well identified components and explored **pairly** relations amongst them

#### components:

elements, phases, processes

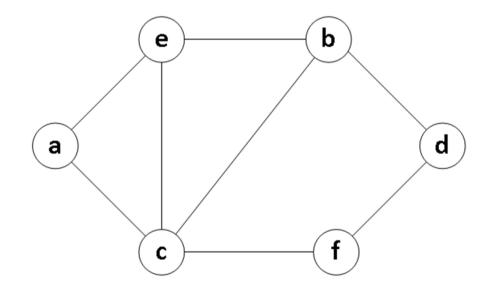
#### relations:

techno-logical dependencies

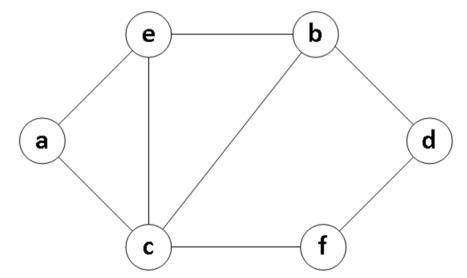
#### mathematically:

structured set of nodes and edges

#### edge: related pair of nodes



Set of nodes:  $N = \{a, b, c, d, e, f\}$  (N=nodes) Set of edges:  $E = [\{a,c\}, \{a,e\}, \{b,c\}, \{b,d\}, \{b,e\}, \{c,e\}, \{c,f\}, \{d,f\}]$  (E=edges) Graph: G = [N, E](G=graph $\cong$  graphics)



#### **Directed Edge ("Arrow"):**

dependency between the two related nodes is interpreted in one direction only

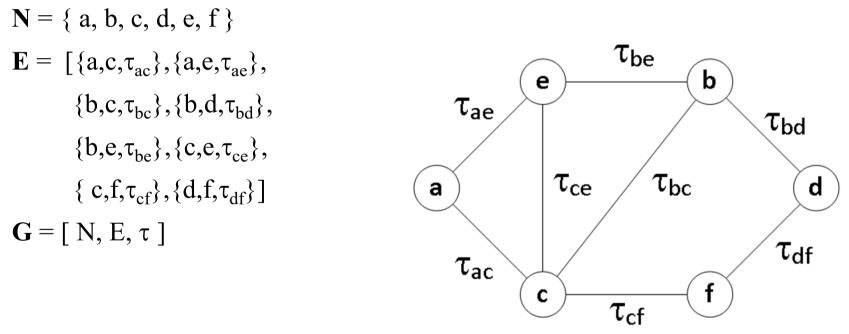
$$N = \{a, b, c, d, e, f\}$$

$$A = \{(a,c),(a,e),(b,c),(b,d),(c,b), (c,f),(e,b),(e,c),(f,d)\}$$

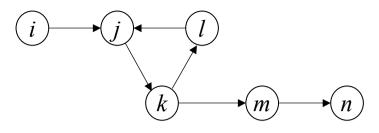
$$G = [N, A]$$
Directed Graph:  
(,,DiGraph "=Directed Graph)  
a graph with all of its edges directed  
But:  $\{i, j\} = \{(i, j), (j, i)\}$ 

#### Weighted Graph:

quantitative characteristics referred as weights are interpreted along the edges



**Directed Wigthed Graph:**  $G = [N, A, \tau]$ 

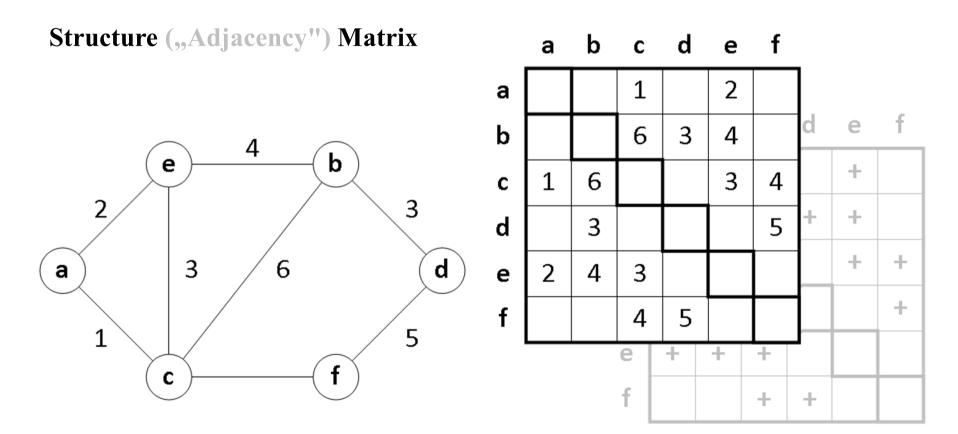


#### **Basic terms of directed graphs:**

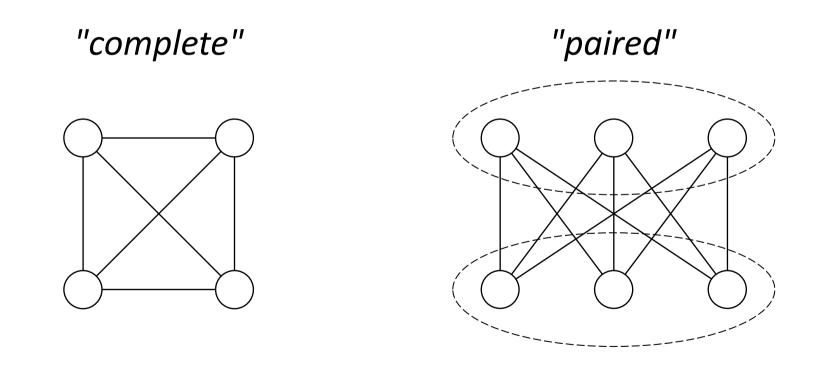
Source: node with at least one leaving arrow but with no entering one e.g.: *i* Sink: node with at least one entering arrow but with no leaving one e.g.: *n* **Route:** series of nodes connected by sequence of directed edges (R=Route) identified by the sequence of nodes involved e.g.:  $R[i,n] = \{i,j,k,l,j,k,m,n\}$ **Path:** sequence of directed edges with no recurring elements (P=Path)e.g.:  $P[i,n] = \{ i, j, k, m, n \}$ identified by the sequence of nodes involved **Loop:** self-closing sequence of directed edges (L=Loop) e.g.:  $L[j,j] = \{ j,k,l,j \}$ identified by the sequence of nodes involved

### **G R A P H**

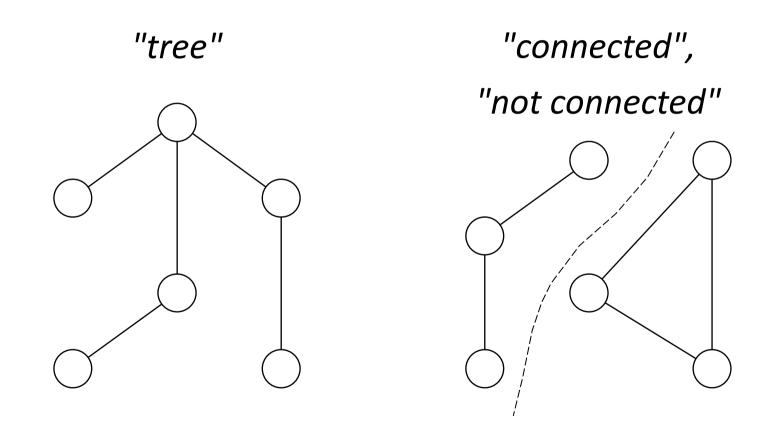
**Length of a Route/Path/Loop:** sum of weights of its edges  $|R_{in}| |P_{in}| |L_{jj}|$ 



#### **GRAPH – topologies** ( relation of nodes and edges/paths )



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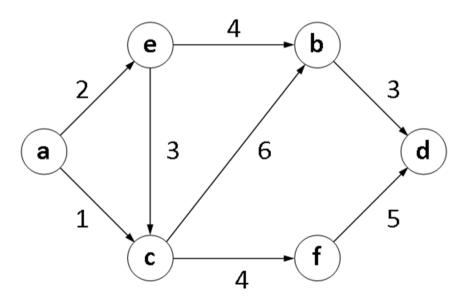
**GRAPH – topologies** ( relation of nodes and edges/paths )

#### **Network:**

(as a special graph topology) connected weighted directed graph, with an only source and with an only sink with no loops and with no negative weights along the edges

#### **Network:**

(*as a synonym of graphs*) graph, in general, with no any restriction on topology or on any other characteristics



### **GRAPH TECHNIQUES – NETWORK TECHNIQUES**

#### **Basic problems (e.g.)**

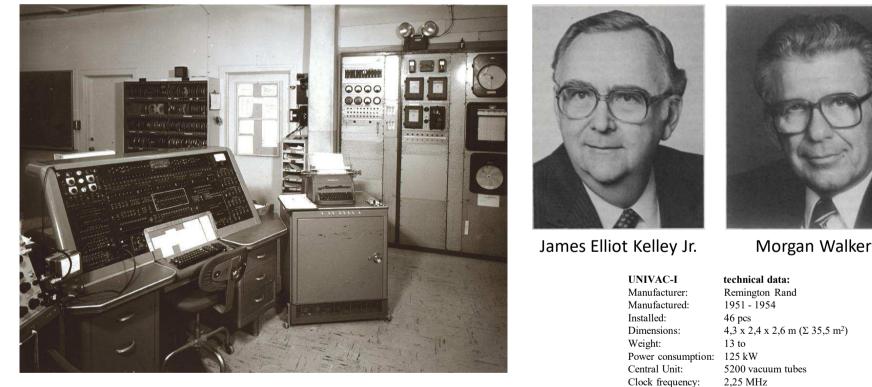
- Path finding
- Connectivity analysis
- Loop discovering
- Dominance analysis
- Path-variants survey
- Longest path / Shortest path
- Gravity point / Centre / Diagonal
- Maximal flow / Minimum cut
- Potentials' problem

#### Analogies in Gaph-techniques

- Longest path problem
- Potentials' problem

#### Scheduling ( Time/Cost Trade-off )

- CPM<sup>time</sup>
- CPM<sup>cost</sup>
- PERT<sup>time</sup>
- CPM<sup>ladder</sup>
- MPM<sup>time</sup> / PDM<sup>time</sup>
- MPM<sup>cost</sup>
- GERT<sup>time</sup>



Remington Rand UNIVAC-I Universal Automatic Computer

Processor speed:

Back-up store:

Input device:

Price:

1905 ops

magnetic tape

punch-card reader

1250-1500 US\$

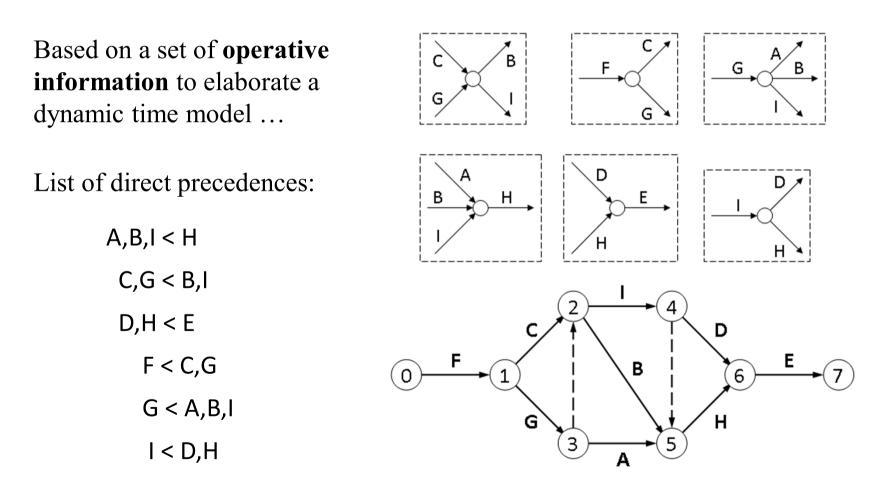
Node: <u>direct</u> succession connection, precedence/sequence/reason-result relation

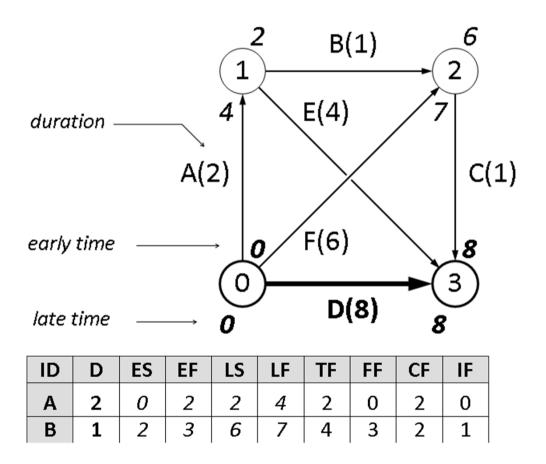
**Edge:** activity/sub-project of well defined technical content, or – by need – relation of direct succession, "dummy activity" (<u>Activity-On-Arrow correspondence</u>)

Edge-parameter (weight): estimated minimum time of performance, "duration" (deterministic – usually integer – variable)

Aim: to calculate intermediate- and overall execution times ("deadlines") of the project;

later: also to identify dominant activity sequences ("critical path") and analysing effects/possibilities of activity delays ("floats") on timing other activities and/or on overall execution time





#### "CRITICAL PATH" (CP):

Subset (subgraph) of a graph composed off nodes – and off <u>dominant edges</u> among them – at which the early and the late times (potentials) equal to each other *(potentials' problem)*;

Subset (subgraph) of a graph composed off the <u>longest paths</u> leading from the source to the sink *(longest path problem)*.

ID=Identifier; D=Duration; ES=Early Start; EF=Early Finish; LS=Late Start; LF=Late Finish; TF=Total Float; FF=Free Float; CF=Conditional Float; IF=Independent Float

"<u>Total Float</u>" (of an activity) : Acceptable increment in duration of an activity (or acceptable delay of its start) with not jeopardizing the <u>early finish of the project</u> assuming that all its (dominant) predecessors can be performed by their <u>early</u> schedules. ("... no delay before, maximum delay after ...")

"<u>Free Float</u>" (of an activity) : Acceptable increment in duration of an activity (or acceptable delay of its start) with not jeopardizing the <u>early schedule of any activity</u> assuming that all its (dominant) predecessors can be performed by their <u>early</u> schedules. ("... no delay before, no delay after ...")

"<u>C</u>onditional <u>F</u>loat" (of an activity) : Acceptable increment in duration of an activity (or acceptable delay of its start) with not jeopardizing the <u>early finish of the project</u> assuming that all its (dominant) predecessors can be performed by their <u>late</u> schedules. ("... maximum delay before, maximum delay after ...")

"Independent Float" (of an activity): Acceptable increment in duration of an activity (or acceptable delay of its start) with not jeopardizing the *early schedule of any activity* assuming that all its (dominant) predecessors can be performed by their *late* schedules. ("... maximum delay before, no delay after ...") (*Non-negative values concerned only !*)

### **Critical Path Method (CPM<sup>cost</sup>)**

#### 1958 : DUPONT de NEMOURS, James E. Kelley - Morgan Walker

Node: <u>direct</u> succession connection, precedence/sequence/reason-result relation

**Edge:** activity/sub-project of well defined technical content, or – by need – relation of direct succession, "dummy activity" (<u>Activity-On-Arrow correspondence</u>)

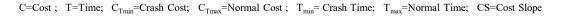
**Edge-parameter (weight):** estimated time-range of performance (variants of duration), with associating estimated direct costs (as function of duration)

**Aim:** to develop an optimal schedule with main consideration of time-cost trade-off at the activities and at the overall execution time of the project ...

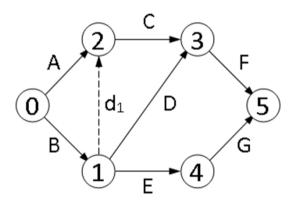
... via iterative steps of calculating intermediate- and overall execution times and changing activity durations (within the pre-set ranges) in a proper way to achieve an optimal solution (of minimum direct costs at a given overall execution time)

#### **Critical Path Method (CPM<sup>cost</sup>)** 1958 : DUPONT de NEMOURS, James E. Kelley - Morgan Walker

**Project Costs** C Indirect **Activity / Sub-project** C Direct **Direct Costs** C<sub>Tmin</sub> ΣΤ C<sub>Tmax</sub> Cost Slope 1 T<sub>min</sub> T<sub>max</sub> Т

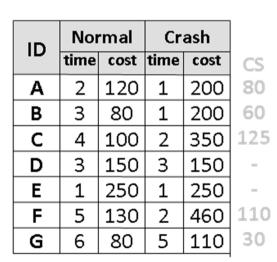


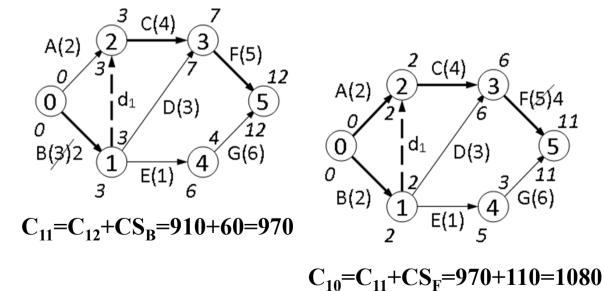
#### **Critical Path Method (CPM<sup>cost</sup>)** 1958 : DUPONT de NEMOURS, James E. Kelley - Morgan Walker



#### (CPM<sup>cost</sup>) Problem:

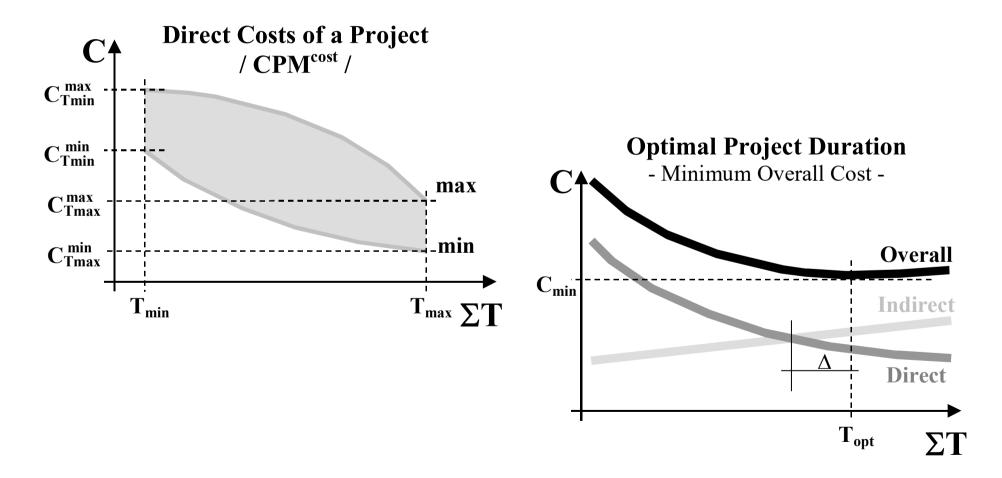
What are the minimum direct costs of the project associating the scheduled overall execution time not longer than 10 time units?

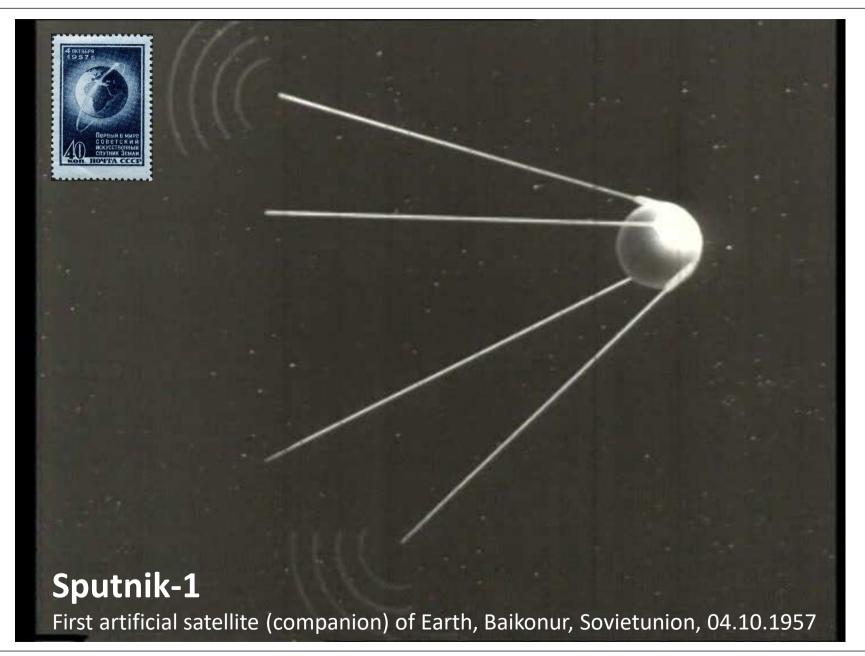




### **Critical Path Method (CPM<sup>cost</sup>)**

1958 : DUPONT de NEMOURS, James E. Kelley - Morgan Walker









#### **US NAVY, Polaris Project**

First successful underwater launch of a missile, USS George Washington 20.07.1960



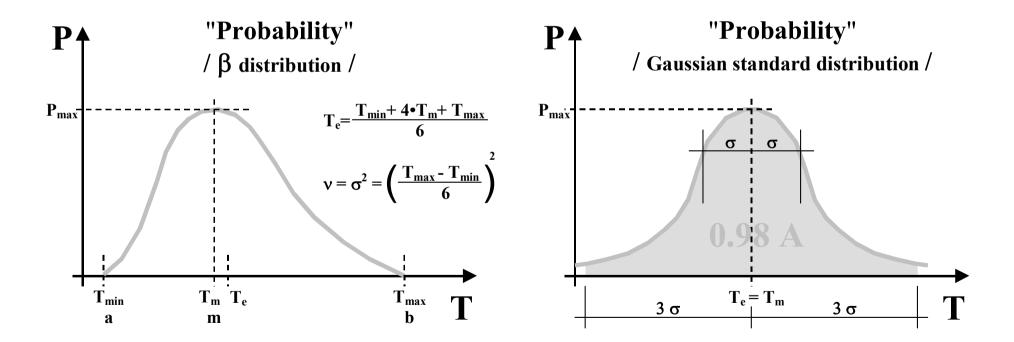
Node: event, state, "mile-stone", phase of progression

Edge: activity ("sub-projects") with closely not identified (technical) contents ("R&D") (Activity-On-Arrow correspondence)

**Parameter (weight):** likely "time-span" of the activity, <u>probabilistic variable</u> of  $\beta$  distribution, predicted by triplex estimates (stochastic model)

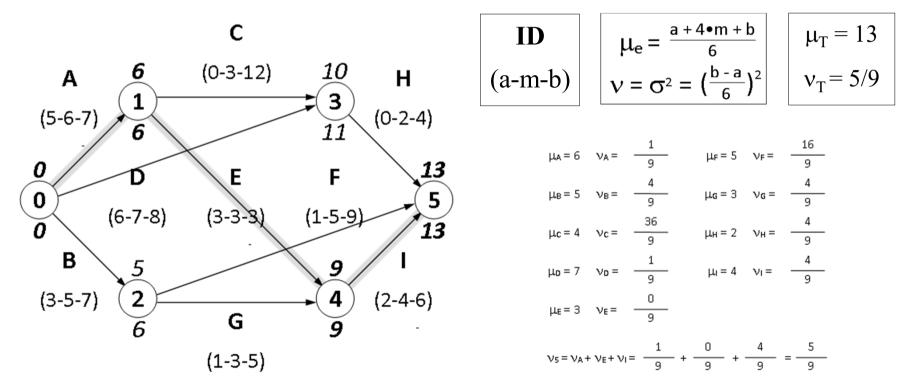
Aim: to predict timing of milestones and overall execution time of the project, together with indices of uncertainty ("deviation"); also to check feasibility of a schedule

Why β distribution? Why 6? : limited range, asymmetry, tolerable error

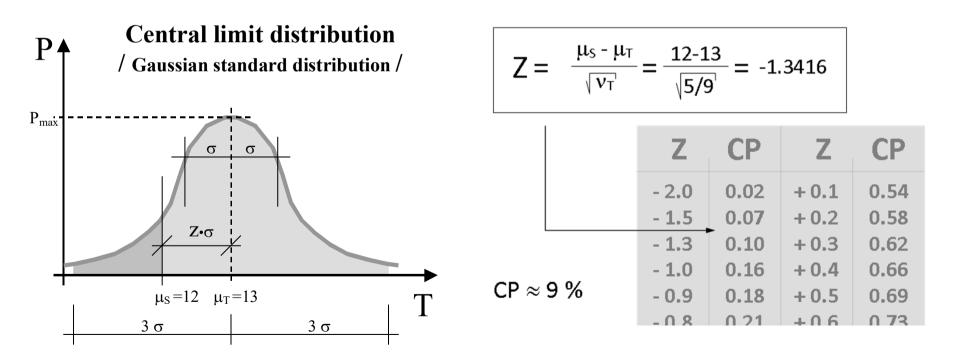


 $T_e = \mu_e$  = expected value;  $T_m = \mu_m$  = modus;  $\nu$  = variance (or: deviation square);  $\sigma$  = deviation; P = propability (density)

**PERT problem :** What is the probability of completing the project below in a scheduled period not longer than 12 time units?



**PERT problem :** What is the probability of completing the project below in a scheduled period not longer than 12 time units?



 $\mu_{s}$  = scheduled time;  $\mu_{T}$  = expected length of the longest path;  $\nu_{T}$  = variance associating  $\mu_{T}$ ; CP = cummulative probability



#### Vostok-1, Yuri Gagarin

On an orbit around the Earth, first man in space, 12.04.1961 09:07 (108 minute mission)



#### **Alan Bartlett Shepard**

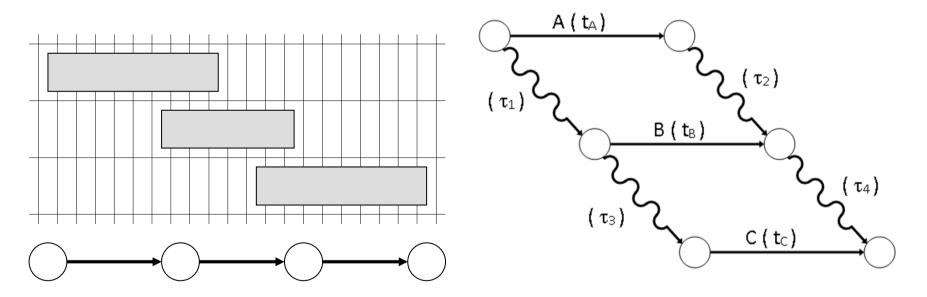
First american in space, on a suborbital trajectory, 05.05.1961(15 minute "space-jump")Fifth american astronaut stepping on the surface of the Moon, Apollo-14, 31.01 - 09.02.1971

#### **CPM**<sup>ladder</sup> Convention

1961..: BTM / ICT, UK, J. Grant, P. A. Rhodes, H. S. Woodgate, ..

**CPM/PERT:** modelling overlapped time positions is unsolved or too complicated

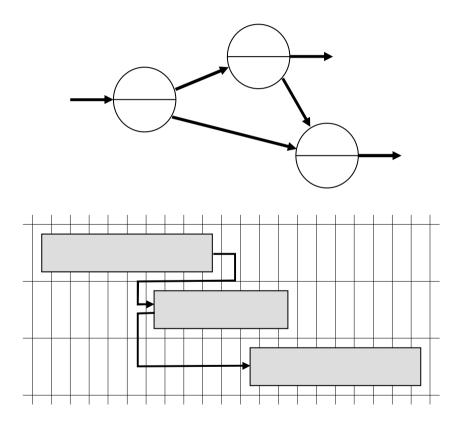
**Proposal:** inserting further dummy- ("ladder" and "hammock") activities



still Activity-On-Arrow ("AOA") correspondence

#### **Precedence Diagramming Method (PDM)**

<u>Activity On N</u>ode correspondence 1962 : Stanford University, John W. Fondahl



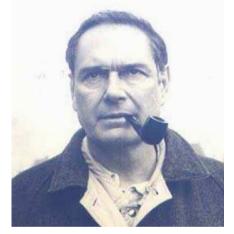


John W. Fondahl

IBM, MicroSoft, "Bar-chart – PERT-chart" projection, USA, 1964-

#### **METRA – Potentials' Method (MPM)** 1958-61 : SEMA-METRA, France, Construction Projects, Bernard Roy





Bernard Roy

Chinon Nuclear Power Plant, Loire valley, France, 1962

### METRA – Potentials' Method (MPM)

#### 1958-61 : SEMA-METRA, France, Construction Projects, Bernard Roy

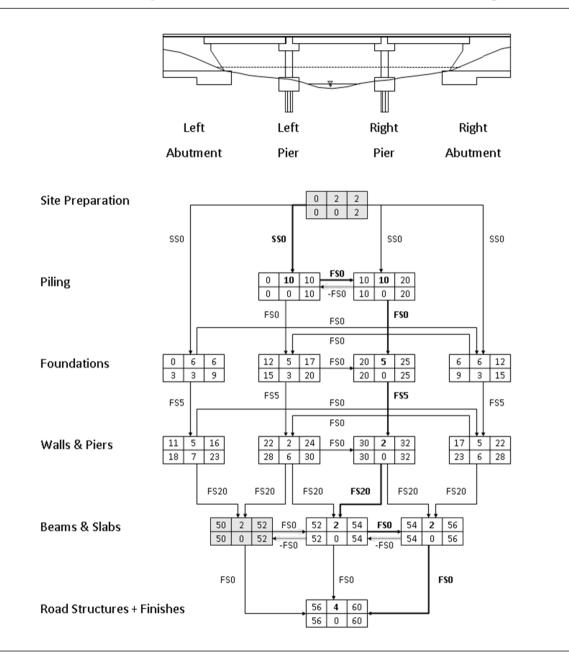
**Node:** activity of fixed duration (0 duration for events or for milestones) (Activity-On-Node correspondence)

Edge: techno-logical or resource-based quantified relation

Parameter (weight): lead- or lag time, duration (usually integer typed deterministic variable)

Aim: modelling technologies, production management, controlling/monitoring, change management ...

... modelling arbitrary relative time positions (e.g. overlapping), lower/upper bounds for timing (minimum/maximum typed relations)...



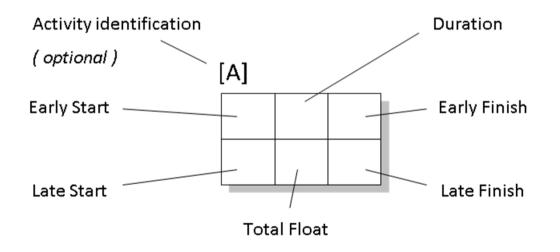
### METRA Potentials' Method (MPM)

1958-61 : SEMA-METRA, France, Construction Projects, Bernard Roy

> MPM time model of the on-site works of a bridge construction project

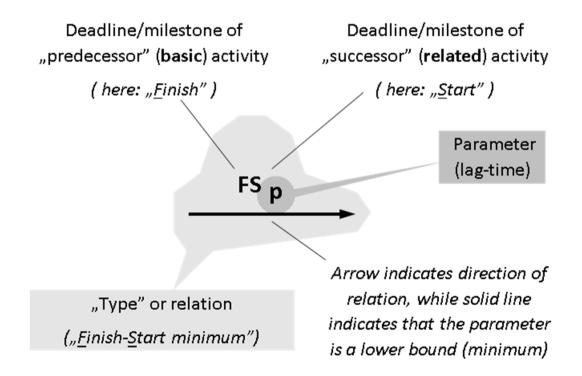
1958-61 : SEMA-METRA, France, Construction Projects, Bernard Roy

Arranging data in an activity box/shield (Node) for manual calculations



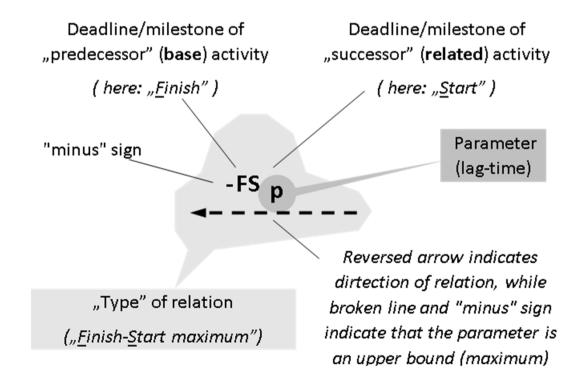
1958-61 : SEMA-METRA, France, Construction Projects, Bernard Roy

#### Graphic representation of lower bound (minimum) typed relations



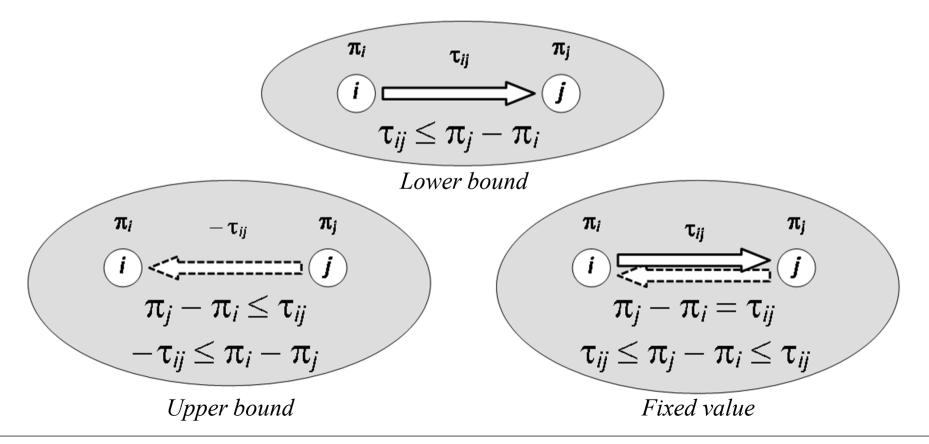
1958-61 : SEMA-METRA, France, Construction Projects, Bernard Roy

#### Graphic representation of upper bound (maximum) typed relations



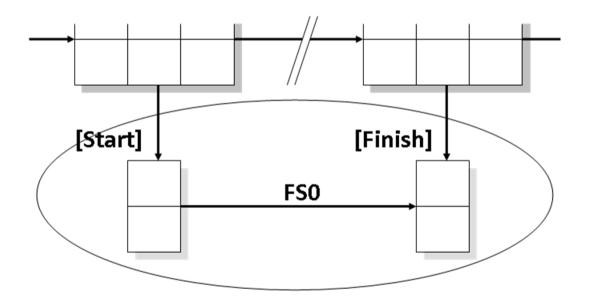
1958-61 : SEMA-METRA, France, Construction Projects, Bernard Roy

"Homogenizing" relations (bounds) – keeping analogy of the "longest path" problem



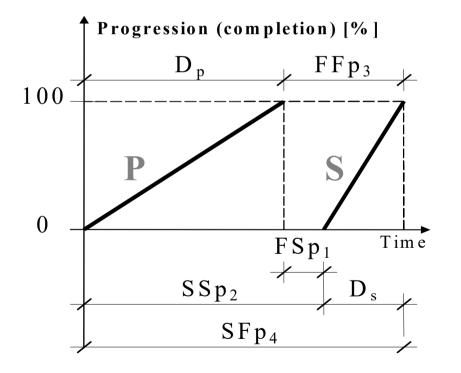
1958-61 : SEMA-METRA, France, Construction Projects, Bernard Roy

"Hammock" ("summary" or "embracing" ) activity



1958-61 : SEMA-METRA, France, Construction Projects, Bernard Roy



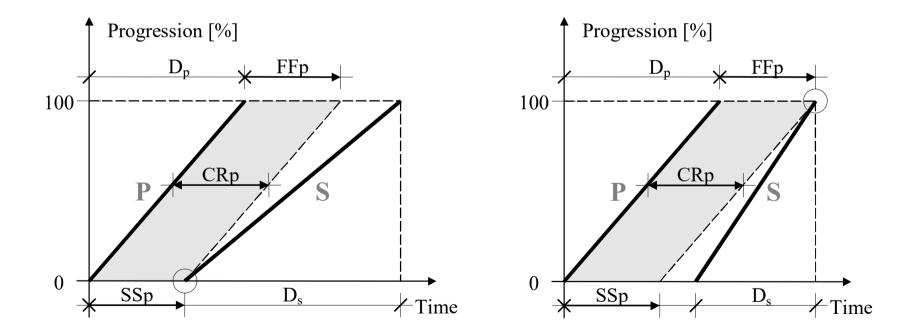


#### **Conversion of basic types of relations**

	FFq	FSq	SFq	SSq
FFp		q = p - D <sub>s</sub>	$\mathbf{q} = \mathbf{p} + \mathbf{D}_{\mathbf{p}}$	$q = p + D_p - D_s$
FSp	q = p + D <sub>s</sub>		$q = p + D_p + D_s$	$\mathbf{q} = \mathbf{p} + \mathbf{D}_{\mathbf{p}}$
SFp	q = p - D <sub>p</sub>	$q = p - D_p - D_s$		q = p - D <sub>s</sub>
SSp	$q = p + D_s - D_p$	q = p - D <sub>p</sub>	q = p + D <sub>s</sub>	

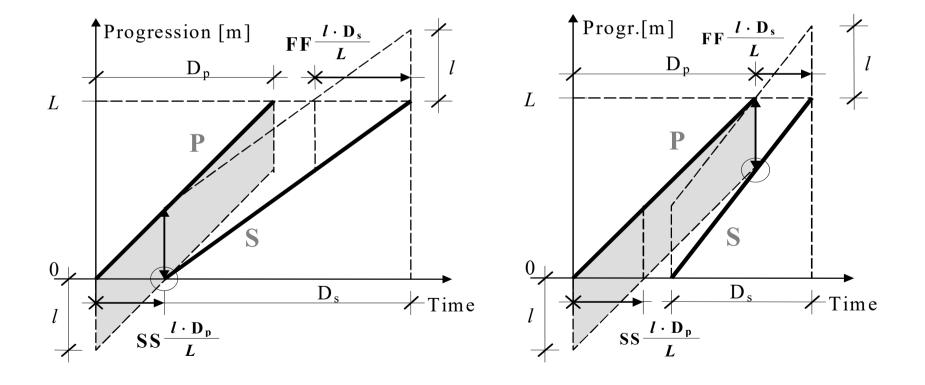
1958-61 : SEMA-METRA, France, Construction Projects, Bernard Roy

Providing (minimum) lead in time ("technological break") between overlapped activities



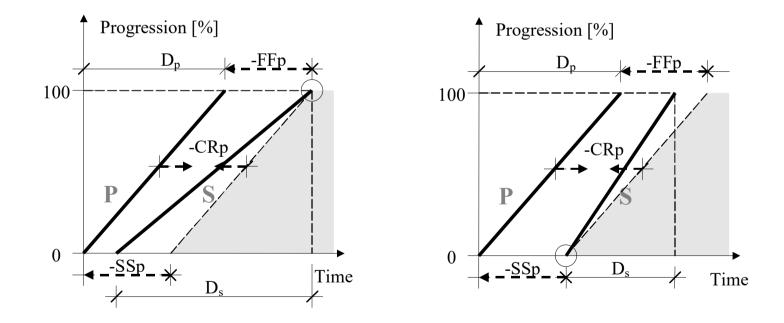
1958-61 : SEMA-METRA, France, Construction Projects, Bernard Roy

#### Providing (minimum) lead in progression ("safety distance") between overlapped activities



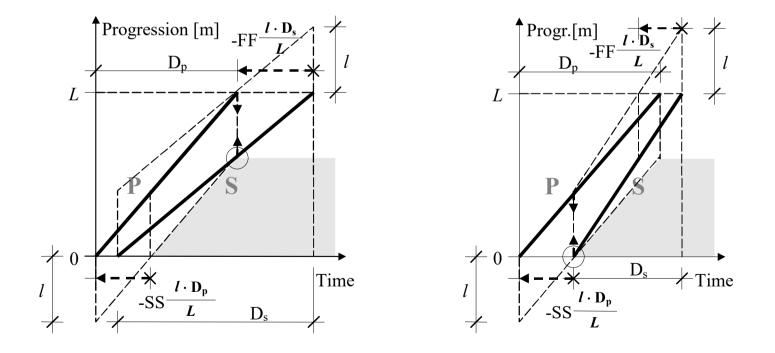
1958-61 : SEMA-METRA, France, Construction Projects, Bernard Roy

Limiting (maximum) lag in time ("sensitive conditions") between overlapped activities



1958-61 : SEMA-METRA, France, Construction Projects, Bernard Roy

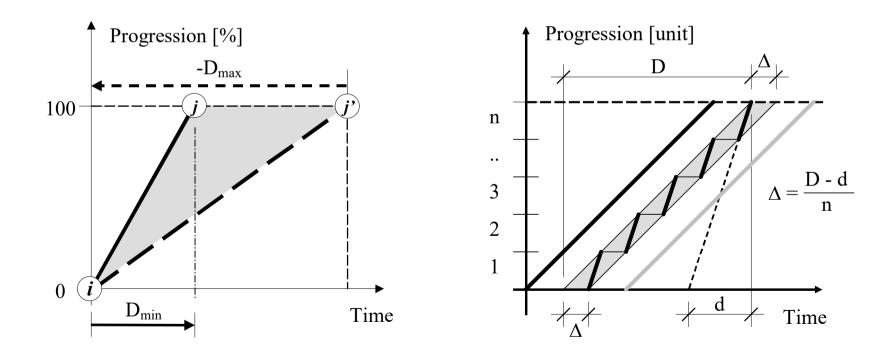
Limiting (maximum) lag in progression ("distance") between overlapped activities



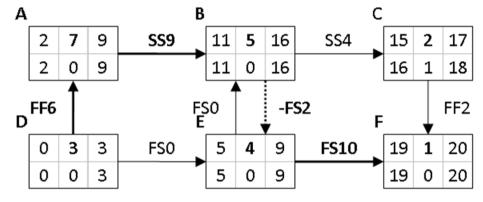
1958-61 : SEMA-METRA, France, Construction Projects, Bernard Roy

**Bounding flexible duration** 

**Resolving duration paradox** 

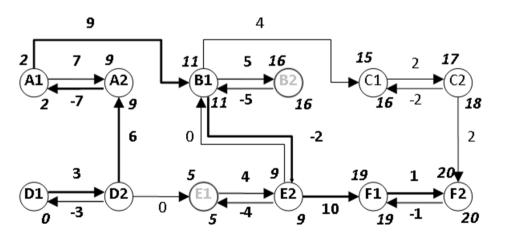


1958-61 : SEMA-METRA, France, Construction Projects, Bernard Roy



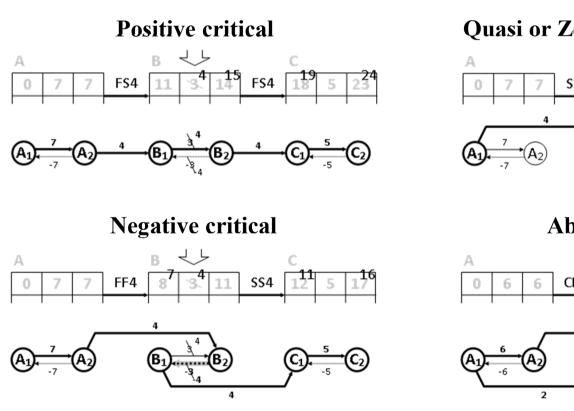
An MPM time model ...

... and its DiGraph equivalent

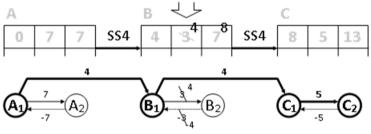


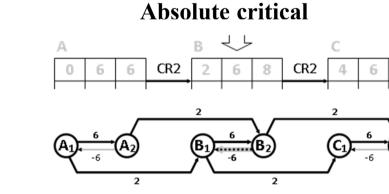
1958-61 : SEMA-METRA, France, Construction Projects, Bernard Roy

### **BEHAVIOUR OF CRITICAL ACTIVITIES (SOME TYPES OF DOMINANCE)**



Quasi or Zero (here: start) critical





### 1958-61 : SEMA-METRA, France, Construction Projects, Bernard Roy

### **COMPREHENSION: FOUR RIDDLES ON NETWORK TIME MODELS**

- 1., Duration of an activity having <u>no float</u> in an <u>activity-on-arrow</u> typed network time model get increased by  $\delta$ . What will be its effect on the overall execution time of the project ?
- 2., Duration of an activity having <u>no float</u> in an <u>activity-on-arrow</u> typed network time model get decreased by  $\delta$ . What will be its effect on the overall execution time of the project ?
- 3., Duration of an activity having <u>no float</u> in an <u>activity-on-node</u> typed network time model get increased by  $\delta$ . What will be its effect on the overall execution time of the project ?
- 4., Do explain a situation when changing the duration of an activity that has no any float in a network time model will surely increase the overall execution time of the project.